GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

Examiners' Reports

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Chief Examiner's Report – Pure Mathematics

Several of the reports which follow mention concerns about the lack of precision in the work of many candidates. Sometimes this refers to the use of notation, accuracy with which is an important aspect of mathematics. The specification booklet includes an appendix listing the notation which might occur and candidates should be aware of those items from that list that might be of relevance to a unit they are sitting. For example, it is often the case that the following are used or interpreted inappropriately:

$$|x|, \quad f^{-1}(x), \quad \sin^{-1}\theta, \quad \frac{dy}{dx}, \quad z^*, \quad \int y\,dx, \quad fg(x) .$$

Lack of precision also occurs with reference to some very basic ideas in mathematics at this level. The following indicate a few of the problems that occur on a regular basis.

1. Candidates often confuse root and factor.

2. Candidates sometimes do not appreciate that, for example, 0.5 radians is not the same as 0.5π radians.

3. Candidates sometimes try to apply the discriminant to equations for which its use is not appropriate.

4. Candidates seldom seem to realise that the existence of an identity such as $\sin^2\theta + \cos^2\theta \equiv 1$ also means that $\sin^2 7\alpha + \cos^2 7\alpha \equiv 1$, $\sin^2 \frac{1}{2}\beta + \cos^2 \frac{1}{2}\beta \equiv 1$ and $\sin^2 17° + \cos^2 17° = 1$ for example.

5. More generally, candidates often fail to appreciate that the existence of an identity means that many particular, related results follow automatically.

6. Candidates often write $2 \times 5y + 3$ for example when they mean $2(5y+3)$. The vital role of brackets in algebra is often not appreciated.

7. Candidates are happy to write $\ln 2 + \ln 3 = \ln 6$, for example, but a lack of care sometimes means that $2\ln 5 + \ln 3$ becomes $2\ln 15$.

8. A common error is the simplification of an expression such as $\sqrt{x^4 + 25}$ to give $x^2 + 5$ …

9. … and the simplification of an expression such as $\dfrac{y^2+10}{y+2}$ to give $y + 5$.

4721 Core Mathematics 1

General Comments

The large majority of candidates were well prepared for this paper and performance was much improved from the January session with a large number of candidates scoring very high marks. Most candidates attempted nearly all of the paper, with only some parts of questions 9 and 10 having significant numbers of omissions.

It was pleasing to see that comparatively few candidates used additional sheets, indicating again that sufficient room was available in the answer booklet for solutions. The only exception was question 9; several candidates tried this on a number of occasions and needed additional paper for their extra attempts. There was a slight issue with several candidates attempting 10ii in the large space beneath the graph for 10i, marks were awarded in these cases.

It was pleasing to see many more candidates use factorisation as their method of choice to solve quadratic equations rather than the more error-prone methods of completing the square or use of the quadratic formula. The fact that this unit does not allow the use of a calculator remains an issue for a large number of candidates; some clearly still need support in dealing with fractions and negative numbers.

Comments on Individual Questions

1) Completing the square continues to improve with very few candidates failing to gain the first mark and the vast majority gaining the first two. Many candidates scored all four marks, the question thus providing a familiar and relatively easy start to the paper. The less successful candidates usually struggled with the combining of the constant terms which they found both arithmetically and conceptually demanding.

2) (i) Most candidates knew the shape of this familiar graph, but the quality of the sketching was extremely variable. Most of the candidates who chose the correct two quadrants but only scored one mark were let down by errors such as not making it clear that the axes were the asymptotes or allowing their curve to move away at several "ends". A few candidates chose the wrong quadrants and some sketched $y = x^2$.

(ii) Incorrect use of language remains a problem describing transformations. Although most candidates recognised that this was a translation, many used "shift", "move", etc. instead of the correct word. Other issues included phrases such as "up/in/on the y-axis" instead of "parallel to the y-axis". A number of weaker candidates thought the translation was in the horizontal direction.

3) (i) Most candidates scored well on this question, securing at least one mark, but there was a significant number of errors in both terms. Many candidates divided both terms in the numerator by x to end with x^3 in their final answer rather than x^4. Others failed to simplify fully, not combining the 16 and 2 to make 32.

(ii) Candidates found this expression much harder to simplify with many seemingly unaware of how to deal with two powers. Most had some idea that the power half meant square root but very few were able to deal with the negative power. Only about a third of candidates secured all three marks, with $-6x$ and $-6/x$ among the common errors.

4) More than half the candidates scored full marks on this question. Nearly all candidates gained the first mark reducing the equations to one variable, almost exclusively choosing to eliminate y, with many doing so completely correctly. Factorisation was the most common approach to solving the resulting equation and was often successful. A significant minority that correctly found the values of x still either forgot to evaluate y or made errors in the arithmetic when doing so, roughly equal numbers having problems substituting the negative or the fractional value of x.

5) (i) This question was very well done, with the vast majority of candidates securing all three marks. A few candidates subtracted the values and then tried to simplify $\sqrt{252}$; some others were unable to evaluate the correct multiple of $\sqrt{3}$ or worked out $\sqrt{16}$ as 8.

(ii) Most candidates realised the need to multiply by $\sqrt{5}$ but it was common to do so only to the numerator which gained no credit. Candidates who did make it past the first step were usually then able to obtain all three marks, although some failed to simplify their answers fully.

6) The vast majority of candidates were able to recognise this disguised quadratic, although some thought that if $y = x^{½}$, $y^2 = x^{¼}$. The resulting quadratic was simple to solve and many did so correctly, but then lost the final two marks by only squaring the solutions at the end rather than raising them to the fourth power.

7) (i) Around three-quarters of candidates scored all three marks here, with only a small number either failing to deal with all three terms in both inequalities or making arithmetical slips.

(ii) A surprising number of candidates did not realise the need to, or were unable to, collect all terms on one side of the inequality sign and thus were unable to make any progress. Those that did this usually started the question well, forming and factorising the correct quadratic. Errors then arose in failing to identify the correct region or eliminating the $x = -2$ part of the solution entirely under the misapprehension "x can't be negative".

8) (i) This question proved to be a good discriminator between candidates. Most candidates recognised the need to differentiate and did so correctly, then setting their result equal to 0; some candidates struggled with the second term, either failing to identify it as $6x^{-1}$ or being unable to deal with differentiating a negative power. A very large number of candidates found solving the resulting equation difficult, with many incorrectly finding $x = 0$ as a solution and still finding a resulting value for y. Many of those that did find $x = -1$ made arithmetical errors in their attempts to find y. Nonetheless, around a third of candidates produced very good, clear solutions scoring all five available marks.

(ii) Most candidates were aware of how to approach this part of the question, most commonly by differentiating again and considering the sign for their value of x. Only around a third of candidates scored both marks, mainly due to errors in the previous part making the follow through marks difficult to obtain.

9) (i) There were many good attempts to this question, with attempts to use the product of gradients or Pythagoras' theorem both frequently seen. More than half of all candidates scored full marks; those who failed to do so either fell down by not fully justifying their choice of A or because of poor arithmetic. The use of a diagrammatic approach did not earn any credit.

(ii) This unstructured seven mark question proved very discriminating, with around a quarter of candidates scoring no marks and many making several attempts. A wide variety of approaches were taken, but the most common was to try to find the mid-point and centre and substitute. Despite having identified A as the right angle in 9(i), a large number of candidates did not make the connection that the mid-point of BC must be the centre and a variety of points were used, including those identified in the question as being on the circumference. Similarly, the value chosen for the radius was often not appropriate and there was confusion between radius and diameter. Some substituted the three points into the given equation and attempted to solve the three simultaneous equations formed; some candidates were successful but other candidates struggled to eliminate one variable.

10) (i) Many candidates sketched the cubic very well. Almost all recognised that it would be cubic with three roots and a large number identified the correct intersections on the x-axis. The intersection on the y-axis was more often omitted than incorrectly evaluated.

(ii) This part proved a rich source of marks to many candidates, with nearly three-quarters scoring all six marks. Errors were more common in the expansion of brackets than in the subsequent differentiation or substitution of $x = 1$; candidates who found a quadratic first and then multiplied by the final linear factor were far more successful than those who tried to multiply all three brackets at once.

(iii) Many candidates were unsure whether to use the same gradient as the curve or its negative reciprocal in finding the equation of the tangent. Many others also had difficulty substituting -2 into the equation of the curve to find the y-coordinate; those who used the factorised form were generally more successful than those who used an expanded version. Most were aware of how to construct the equation of a line, but many marks were lost for the reasons just described or for finding y erroneously from the gradient function.

(iv) More than half the candidates failed to score on this part, with many making no attempt, which was rare for this paper. Candidate who substituted -2 into the gradient function were the most likely to succeed, but many again showed confusion as to whether the line should be parallel or perpendicular. Equating the equations of the line and the curve very seldom led to full marks.

4722 Core Mathematics 2

General Comments

The paper was accessible to most candidates, and it allowed them to demonstrate a sound understanding of the topics being tested. Some candidates lost marks through a lack of algebraic fluency when evaluating expressions or rearranging equations. Lack of brackets also resulted in candidates misinterpreting their own intentions and evaluating expressions incorrectly.

Whilst many candidates set out their solutions in a clear and concise manner, this is not always the case. In some cases a lack of clarity can result in candidates misreading their own work, and in other cases the examiner may be unable to give credit if the working is ambiguous. This was particularly noticeable when using the quadratic formula in that the original statement would be correct, but the fraction line would then shorten to the extent that the final answer would be incorrect.

Some candidates are ignoring the advice given about multiple attempts. Unless they indicate otherwise, it is the final attempt that will be marked even if this gains less credit than other attempts so candidates are strongly advised to delete all but the solution that they wish to be marked.

Candidates seem to be paying more attention to accuracy, both in their working and also in giving answers to the requested number of significant figures. However, marks are still being lost through giving exact answers in decimal form instead.

This was the third time that C2 was answered using a booklet, and the majority of candidates coped well with this. Solutions were given in the correct answer space and, where errors were made, candidates often wrote explanatory notes in the margin. There were some problems in reading the script where candidates had worked in pencil and then gone over it again, or rubbed out answers and written over the top. Candidates are also advised not to write in black felt-tip pen as this shows through onto the reverse of the page and makes it more difficult to read the solution that is there.

Comments on Individual Questions

1) (i) This proved to be a straightforward start to the paper, with the majority of candidates gaining both of the marks available. When marks were lost this was usually through failing to appreciate the order of operations required in the cosine rule and actually evaluating $(b^2 + c^2 - 2bc)\cos 40°$. Other errors included having the calculator in radian mode, or using sin40° rather than cos40°.

(ii) This question was also very well answered, with the majority of candidates able to quote the correct formula and then apply it accurately. A few omitted the ½ and others used more cumbersome methods which sometimes resulted in a loss of accuracy in the final answer.

(iii) Whilst most candidates seemed familiar with the sine rule a number struggled to apply it correctly to this question. The most common error was to find the length of *AD* rather than *BD*, or to incorrectly place the given angle of 63°. Candidates who drew a sketch diagram tended to be more successful than those who did not do so.

2) (i) The integration was carried out correctly by most candidates, although a few struggled to simplify the coefficient correctly. A small minority spoiled an otherwise correct solution by leaving d*x* or an integral sign in their final answer.

(ii) This part of the question proved to be more challenging and a number of candidates attempted to use $y = mx + c$, or used the values of 4 and 17 as limits in a definite integration. Many candidates however did appreciate what was required and could make a good attempt at finding the required equation. Evaluating $4 \times 4^{1.5}$ caused problems for some, and others ignored the fact that an equation was required and omitted '$y =$' .

3) (i) A pleasing number of fully correct solutions were seen to this question, employing clear and concise methods. A few candidates seem to mistakenly believe that π is the symbol for radian measure and, having obtained 0.9, went on to give the final answer as 0.9π which was penalised. Others seemed unsure as to what units they were using and attempted to convert 0.9 into radians using a factor of $^{\pi}/_{180}$. Some candidates could correctly state that the arc length was 8θ, but then equated this to the entire perimeter rather than first finding the arc length. Others attempted to work initially in degrees and then convert into radians, but this was rarely successful.

(ii) Most candidates could use their angle in the correct formula for the area of sector, thus gaining at least one mark.

4) (i) Most candidates appreciated what was required, but many were unable to correctly change the subject of the formula. The most common errors were to square term by term, or to rewrite $\sqrt{(x + 4)}$ as $\sqrt{x} + 2$. Whilst a number of fully correct solutions were seen, it was disappointing that these were not more frequent.

(ii) The integration itself was usually carried out well, although it was quite common to see the third term as $3x$ not $3y$. Most candidates could then attempt F(3) - F(1), but the second term being negative caused problems for some. However, a significant number of candidates did not appreciate that they had found the required area and went on to use it in a further calculation which often involved subtracting their answer from the area of a rectangle. Others attempted a subtraction prior to integrating. Some candidates decided to ignore the 'hence' instruction in the question and attempted to find the area between the curve and the x-axis before subtracting from the area of the rectangle. This required the use of a C3 technique and was given full credit if done correctly but this was rarely the case.

5) (i) The vast majority of candidates were able to correctly state 243 as the value of a, although a significant minority gave the answer as 3.

(ii) This part of the question proved to be much more challenging. The majority of candidates could make an attempt at a binomial expansion, usually gaining a mark for getting a correct second term. The majority then went on to gain a method mark for attempting to find the third term, but the answer was frequently given as $270kx^2$. The most successful candidates made effective use of brackets, but it was not uncommon to see an incorrect term resulting from a previously correct statement involving brackets. Some candidates then equated the terms rather than the coefficients, and produced a solution that still involved x, which gained no credit. A few candidates opted to first write $(3 + kx)$ as $3(1 + {}^{kx}/_{3})$, but this was rarely successful.

(iii) Candidates who had been successful in part (ii) usually went on to score full marks in this part of the question as well, but a lack of care when cubing the kx meant that $90k$ was an all-too-common error.

6) (i) The majority of candidates was able to correctly apply the factor theorem. However, some candidates seemed unfamiliar with the terminology and stated that the factor was $x = 2$, or even f(2).

(ii) Most candidates were able to make a good attempt at this part of the question, with many obtaining the correct quadratic factor. Polynomial division and inspection tended to be the most accurate methods, with more errors resulting from coefficient matching. The majority of candidates could then attempt to find the roots of the quadratic, either through using the formula or completing the square. Common errors included giving decimal not exact roots and giving factors rather than roots. A number of candidates omitted to give 2 as a root.

7) (a)(i) The vast majority of candidates could attempt to use the correct expression for the n^{th} term of a GP but a lack of care when evaluating meant that -1792 was a common error.

(ii) Candidates tended to be more successful in this part of the question, with many fully correct solutions seen. However there were still some candidates who were unable to evaluate a correct expression, and others used r^{n-1} rather than r^n in their expression.

(b) Whilst a number of fully correct solutions were seen, many candidates struggled with this question. Most could state a correct expression for S_N but struggled with the algebraic manipulation which was then required. Expanding $(n - 1)d$ with a negative d caused problems for many with $(n - 1)$ -2 often becoming $n - 3$, or only one term in the bracket being multiplied by -2. Candidates usually then equated their expression to -2900 but many then struggled to rearrange the equation to a form that could be solved. Dealing with the ½N caused problems and it was quite common to see -5800 appear but with the other side given as $2N(2a + (n - 1)d)$, and not all terms were multiplied by N when expanding the bracket. Given that this resulted in a quadratic, it was surprising how many candidates failed to gather all of the terms on one side before attempting to solve. Those who obtained the correct quadratic usually went on to solve it successfully, though not all appreciated the need to discard $N = -50$.

8) (i) Many candidates clearly understood what the transformation did but were unable to use the correct mathematical language to describe it. Any term other than translation was penalised, and for the second mark examiners expected to see a clear and unambiguous indication of 3 units in the negative y direction.

(ii) The more able candidates were able to correctly state -2, either from knowledge of the graph or by evaluating $2^0 - 3$. The most common wrong answer was -3, but it was also quite common to see 2^0 become either 0 or 2.

(iii) Candidates with a good understanding of indices and logarithms simply rearranged the equation to $2^x = 3$ and then deduced that $x = \log_2 3$. Other candidates introduced logs on both sides, using a base of 2 either immediately or subsequently, to obtain the required answer. However many candidates employed more routine methods to obtain ${}^{\log 3}/_{\log 2}$ but could not then make further progress. Any correct expression, including 1.58, could gain one of the two marks available. Given the number of candidates who used log base 2 in the next part of the question it was surprising not to see more fully correct solutions in this part.

(iv) A number of fully correct solutions were seen, possibly because this is a more routine question. Some candidates went straight to $p = \log_2 65$ and others used logs on both sides with both methods being equally successful. The most common error was to introduce logarithms before rearranging.

(v) Candidates are becoming more proficient in applying the trapezium rule and, whilst the usual errors of using x values or attempting integration first were seen, a pleasing number of fully correct solutions were also seen. Once again, a lack of brackets when evaluating the expression caused problems for some.

9) (a)(i) This was probably the most poorly answered question on the paper. Many candidates seemed unfamiliar with the term 'period' and gave the answer as an inequality or even tried to describe a transformation. A number of candidates did state π as the period, and 180° was also accepted, but 2π and 4π were also common.

(ii) Candidates also struggled with this part of the question, and fully correct solutions were relatively rare. There was more success in identifying the y-coordinate than the x-coordinate. Some spoilt a correct answer by reversing the coordinates and others by giving a value in degrees rather than radians.

(iii) The majority of candidates was able to attempt the correct solution method to find at least one angle, which was accepted in degrees, or exact or decimal radians. Finding the second angle caused more problems as candidates struggled to determine what to subtract their first value from. Whilst a number of candidates did manage to produce the two required angles, and no extras, very few then even attempted an inequality with even fewer doing so correctly.

(b) Whilst most candidates seemed familiar with the tan identity, many struggled to apply it accurately to this question. A common error was for $^{\sin 2x}/_{\cos 2x}$ to become $\tan x$. Even those who obtained $\tan 2x$ often equated it to $\sqrt{3}$ rather than the correct $^{1}/_{\sqrt{3}}$. However, these candidates could still gain a mark if they then employed a correct solution method. Some candidates decided to square both sides, but this was rarely successful with a common error being to also square the $2x$. It was also quite common to see only one side of the equation being squared, with an identity then being used to produce a three term quadratic.

4723 Core Mathematics 3

General Comments

This paper produced the usual wide range of responses with each mark from 0 to 72 being recorded. Examiners reported that there were fewer scripts with very low marks on this occasion although approximately 1.5% of candidates recorded 10 marks or fewer. On a paper with a fair number of very accessible marks for dealing with routine requests, this does mean that a number of candidates had made very little progress with their study of mathematics at this level. At the same time, it is pleasing to acknowledge the mathematical ability demonstrated by those candidates scoring very high marks; a sizeable group scored full marks and over 2% of the candidates recorded 70 marks or more.

At this level, the wording of questions is bound to differ somewhat from session to session even when the mathematics being assessed is similar. Part of the test for candidates is to judge carefully what is required of them. In this paper, many candidates seemed not to do themselves justice by their solutions to question 2; the techniques being assessed should have been familiar ones but many candidates were unable to construct convincing solutions. Similarly in question 8(ii) where the specification topic of exponential growth was assessed, many candidates seemed to give little or no thought as to what an appropriate line of action would be and struggled to record any marks.

It may seem a small point but more familiarity with Greek letters would help some candidates. In question 9(i), a proof was required in which the letters θ and α were involved. Proving a given result does need meticulous work and candidates who have difficulty forming the two Greek letters clearly are going to struggle to provide a convincing proof.

Comments on Individual Questions

1) Part (i) was generally answered accurately and there were only a few instances of the coefficient of e^{2x+1} becoming 12 or remaining as 6. Success was not quite so widespread in part (ii) and the answer $10\ln(2x+1)$ was sometimes seen. There were also some instances of answers involving $(2x+1)^0$ or $(2x+1)^{-2}$. A correct answer involving brackets rather than modulus signs was allowed on this occasion, but it was disappointing to see answers such as $5\ln 2x+1$ with neither brackets nor modulus signs used. One easy mark was available for at least one appearance of the constant of integration; a considerable number of candidates failed to earn that mark.

2) Responses to this question were often poor and only about 25% of candidates recorded full marks. There were errors associated with all three of the transformations. $y=\ln(-x)$ was often noted as the result of the reflection. Applying the stretch led to equations such as $y=-\ln(3x)$ and $y=-\ln(\frac{1}{3}x)$ nearly as often as the correct $y=-3\ln(x)$. Some candidates misread the question and attempted to translate by 4 rather than by $\ln 4$.

   The request to give the final equation in a particular form was ignored by a significant number of candidates. Many others were unable to apply logarithm properties correctly and errors such as $-3\ln x+\ln 4=-3\ln 4x$ were common. A few candidates were too concerned from the outset about the form required for the final answer and equations such as $y=-3\ln(\mathrm{f}(x))+\ln 4$ were sometimes seen.

3) (a) This equation caused few problems and approximately 82% of candidates found the correct value of $\cos\alpha$. There were a few errors resulting from incorrect removal of the brackets from $7(2\sin\alpha\cos\alpha)$ and a few candidates tried to proceed without any use of the double-angle identity. Many candidates proceeded to find the value of α but there was no penalty for doing so.

3) (b) The identity giving $\cos 2\beta$ in terms of $\cos\beta$ was not always known and many candidates starting with $\cos 2\beta \equiv \cos^2\beta - \sin^2\beta$ made careless slips in dealing with $\sin^2\beta$. Sometimes these slips meant that the equation to be solved became linear in $\cos\beta$ but, otherwise, the quadratic equation was solved well. The link between cosine and secant was known to most candidates but it was common for two values, $-\frac{3}{2}$ and $-\frac{2}{5}$, to be offered as the final answer. No reason for rejecting the latter value was required but candidates giving both values did not earn the fifth mark. About half of the candidates earned all five marks.

4) There was a disappointing response to the apparently straightforward request in part (i). The graph of $y = (x-2)^4$ did not always extend as far as the y-axis and, in some cases, the graph of $y = x+16$ was a straight line parallel to the x-axis. Many candidates did not note that the two graphs have a point of intersection on the y-axis. A significant number of candidates merely drew the two correct graphs. For the third mark, not much was needed but attention did need to be drawn to the two intersections.

For candidates with careful graphs in part (i) and an understanding of what the term 'root' means in this context, part (ii) represented an easy mark. But that understanding was not present in many cases and it was common for the answer (0, 16) to be given or for an answer to consist of '(0, 16) $x = 0 \quad y = 16$', indicating that the term 'root' was not understood. Only 55% of candidates earned the mark for part (ii).

Part (iii) was answered well and 80% of candidates recorded full marks. Candidates duly recorded details as expected and the only errors to occur with any frequency were final answers of 4.12 or 4.117 instead of 4.118.

5) This question required knowledge of various differentiation techniques, care in setting out the solution and attention to detail. Use of the product rule and quotient rule was generally sound but candidates were less accurate when applying the chain rule; it was often the case that $\ln(4x-3)$ was differentiated to give $\dfrac{1}{4x-3}$.

Most candidates applied the product rule to find the first derivative although the details were sometimes wrong, as indicated above or when the term $\dfrac{4x^2}{4x-3}$ was simplified to $\dfrac{x^2}{x-3}$. Some candidates then saw no further need to use product or quotient rules when finding the second derivative and, as a consequence, no further marks could be awarded. However, most candidates did attempt further application of differentiation techniques as appropriate although the details were again wrong in some cases.

Successful candidates tended to be those who presented clear, concise solutions taking just a few lines of working and which showed that they were thoroughly familiar with the techniques required. Candidates who felt the need at each stage to define this function as u and that function as v and to break the solution into different parts had much more scope for error.

Candidates who had negotiated the differentiation accurately generally substituted correctly to earn the final marks although there were a few cases where the final answer was not simplified sufficiently.

6) The vast majority of candidates earned some marks on this question but the unstructured nature of the question meant that some found it a challenge to deal successfully with all aspects. Most candidates earned the two marks for correct differentiation of $(3x-5)^{\frac{1}{2}}$ and, later, two more marks for correct integration to obtain $\frac{2}{9}(3x-5)^{\frac{3}{2}}$. Earning further marks required some clarity of thought as to what was required and the adoption of an appropriate strategy.

There are various methods of showing that the x-coordinate of P is $\frac{10}{3}$ (some not including differentiation at all) but many candidates made very little progress with this part of the question. The most successful method adopted by candidates was to solve the equation obtained by equating the general expression for the gradient of OP to the derivative, i.e. to solve $\frac{\sqrt{3x-5}}{x}=\frac{3}{2}(3x-5)^{-\frac{1}{2}}$. Other approaches were attempted and there was success in some cases.

Candidates seemed to be on more familiar ground when trying to find the area although various errors were commonplace. The limits for the integration often appeared as 0 and $\frac{10}{3}$; substitution of the limit 0 was sometimes assumed to result in a value of 0 and sometimes the resulting term involving $(-5)^{\frac{3}{2}}$ was ignored. It was surprising how many candidates used integration for finding the area of the relevant triangle, but the equation of the straight line used for this was sometimes wrong with the equation $y = x$ being used in some cases.

An alternative approach to finding the area was pursued by some candidates successfully. This involved rearranging the equation to $x=\frac{1}{3}y^2+\frac{5}{3}$ and finding the area between the curve and the y-axis.

7) This question was handled well by many candidates, approximately 40% of whom recorded all eight marks. The attempts at part (i) did reveal that many candidates did not know the basic definition of $|x|$ for a particular value of x and there were many attempts at solving two linear equations, one with a right-hand side of 12 and the other with a right-hand side of –12. Other common attempts involved squaring both sides, or in a few cases just one side, of an equation. Candidates with a clear understanding were able immediately to write $3(x+2)+5=12$ and proceed to the correct answer without fuss.

Part (ii) was answered very well. There were a few candidates who thought $\text{gg}(x)$ meant $(3x+5)(3x+5)$ and a few more for whom the notation $\text{h}^{-1}(x)$ prompted differentiation or the formation of a reciprocal but, otherwise, only occasional algebraic slips or a failure to present the final answer in an acceptable form prevented the award of all three marks.

It had been anticipated that part (iii) might present problems to candidates but, in the event, approximately 70% of candidates gave the correct answer $x \le 0$, or the answer $x < 0$ which earned one mark. Many candidates presumably made sensible use of their graphical calculators to reach the correct conclusion. Some candidates, having experimented with a few particular values, merely offered answers such as $x=-1, -2$. Others, prompted by the presence of $|x|$, resorted to squaring and then offered only $x = 0$ as the answer.

8) The majority of candidates had no difficulty with part (i), differentiating accurately and giving the answer to an appropriate degree of accuracy. Examiners were tolerant of the presence or absence of the minus sign in final answers. A few candidates substituted the value 10 before attempting differentiation and some found an average rate of change over the interval from $t = 0$ to $t = 10$.

Part (ii) presented more problems to candidates although almost half did record at least three marks. Many candidates did not appreciate that they had to find a formula for M_2 and there were attempts at equating $400\text{e}^{-0.014t}$ to 75 or 120. For those attempting to find a formula, some started with an incorrect form such as $74+\text{e}^{kt}$. A few candidates found the correct formula $M_2 = 75\times 1.6^{0.1t}$ but this was a form which presented major difficulties when trying to reach an equation of the form $\text{e}^{kt} = c$. Candidates who had managed to find the formula $M_2 = 75\text{e}^{0.047t}$ were often unable to follow this with a correct rearrangement of $400\text{e}^{-0.014t} = 75\text{e}^{0.047t}$ to a form in which e appears just once.

Inevitably the problems with part (ii) meant that two marks in part (iii) were earned by only a quarter of the candidates although one mark was available to candidates demonstrating the correct method for the solution of an equation of the form $\text{e}^{kt} = c$.

9) Some progress was made in part (i) by the vast majority of candidates and most were able to apply the relevant identities to reach the simplified version $\dfrac{2\sin\theta\cos\alpha + 3\sin\theta}{2\cos\theta\cos\alpha + 3\cos\theta}$ although some were guilty of a lack of attention to the detail. However, the subsequent simplification was not done well and many candidates carried out all sorts of invalid cancellation procedures in their endeavours to reach $\tan\theta$.

In tackling part (ii), a significant number of candidates forgot all about part (i) and resorted to a calculator exercise to find the (approximate) value of the expression. Many others did try to establish a link with the identity from part (i) although some decided $\tan 150°$, $4\tan 150°$ or $\tan 450°$ were appropriate. Others did reach the correct $\frac{4}{3}\tan 150°$ although not all concluded by giving this in correct surd form.

Part (iii) defeated many candidates who made little or no relevant progress towards a solution. Nearly 60% of candidates did take the promising first step of stating $\tan 6\theta = k$ but many then complicated matters by trying to use identities apparently to attempt to express $\tan 6\theta$ in terms of $\tan\theta$, a solution that was typically abandoned after a page or so of haphazard trigonometry. About a quarter of the candidates did recognise that $\tan 6\theta = k$ led immediately to $\theta = \frac{1}{6}\tan^{-1} k$ but it was only a small number who realised that there was a second answer to be given. Doubtless an equation such as $\tan 6\theta = 5$ would have been solved correctly by far more candidates but, at this level, candidates should be comfortable dealing with the slightly more abstract $\tan 6\theta = k$.

4724 Core Mathematics 4

General Comments

This was, possibly, a harder paper than normal but only slightly so. Some presentation still needs to be improved and considerable portions of the work were difficult to read. As usual, explanations are often not clear but they are vital when wrong answers are obtained.

Having said that, there was much work of a very high standard and these candidates are to be congratulated; it is still remarkable though that 40 candidates scored 0 or 1 and that $2\frac{1}{2}$% of the entry scored 10 or less.

Candidates are not happy giving explanations, as in Qu.9(ii)(b); the intention was that they should explain how each part of the given formula was implicated but most failed quite miserably. No doubt there will be some improvement if, in future, more questions of this type are set.

Time did not seem an issue.

Comments on Individual Questions

1) Although a few candidates attempted the principle of partial fractions, the majority realised that factorisation was the best way forward. However, the simplest factorisations were often neglected with a number discovering that the numerator was made up of $\left(x^2-2x-3\right)\left(x^2+2x-3\right)$. The simplest way was to factorise the numerator as $\left(x^2-9\right)\left(x^2-1\right)$ and then $(x-3)(x+3)(x-1)(x+1)$ whilst the denominator could be expressed as $(x-3)(x+1)(x+3)(x+5)$. Subsequent cancellation reduced the fraction to $\dfrac{x-1}{x+5}$.

2) Although the concept of a unit vector had been examined fairly recently, the idea was still not fully understood. Most realised they had to evaluate $2^2+(-3)^2+\left(\sqrt{12}\right)^2$, a fair majority then taking the square root. What to do with the '5' was not understood by most and only relatively few answers of $\frac{1}{5}\begin{pmatrix}2\\-3\\\sqrt{12}\end{pmatrix}$ were seen. A number of students evaluated $2^2-3^2+\left(\sqrt{12}\right)^2$; these were credited with knowing basically what to do and, if they managed to reach $\sqrt{25}=5$, their lack of 'clarity' was overlooked and they received further credit.

3) (i) As has often been stated in the past, straight forward long division was the easiest method for this part – though the use of an identity was much improved from previous series. The only danger here was that, having been asked to show that the remainder was x, a few wrote the RHS as $(ax+b)\left(x^2+3\right)+cx$.

(ii) The candidates were divided into two camps here, those who said they had to integrate, typically, $3x-1+\frac{x}{x^2+3}$ and those who integrated $3x-1+x$. $\int \frac{x}{x^2+3}\,dx$ proved the only stumbling block though most knew that $\ln\left(x^2+3\right)$ was involved; its multiple of $\frac{1}{2}$ was generally, but by no means always, known.

4) In only a relatively few cases was the 'dx' just changed to '$d\theta$' or not altered at all. The first key aspect was the conversion of the denominator to $\cos^3\theta$, which many did not manage; $9x^2$ was often transformed into either $(3\sin\theta)^2$ or $3\sin^2\theta$ and, not infrequently, the power $\frac{3}{2}$ which candidates correctly transcribed on line 1 was dropped at this stage. The integration of $\frac{1}{3\cos^2\theta}$ often proceeded in the correct direction (though with the '$\frac{1}{3}$' often becoming '3') as $\frac{1}{\cos^2\theta}$ was transformed into $\sec^2\theta$, an aspect not penalised until the final answer was marked. However, many candidates remembered that $\cos^2\theta$ had a connection with $\cos 2\theta$ and increasingly poor integration/algebra followed.

5) (i) Almost every candidate coped with this, with some less systematic than others.

(ii) There was clear understanding of both scalar product and modulus; the only problem was deciding which vectors to use.

(iii) The correct solution to this type of problem is now seen more often, though it was answered much less successfully than part (ii).

6) Various manipulations were used to obtain the required coefficient; the least successful were the changes into $\frac{(1+ax)^{\frac{1}{2}}}{(4-x)^{\frac{1}{2}}}$ or $\left[(1+ax)\left(4-x^{-1}\right)\right]^{\frac{1}{2}}$, the first because of division and the second because of the awkwardness of the expansion of an expression of the type $\left(p+qx+rx^2\right)^{\frac{1}{2}}$. The majority of candidates chose the most suitable one in the context, that of $(1+ax)^{\frac{1}{2}}(4-x)^{-\frac{1}{2}}$. Whichever method was chosen, ability to deal with rational/negative powers was tested. Two problems, in general, affected the overall result: how to deal with $(4-x)^k$, where k could be $\frac{1}{2}$ or $-\frac{1}{2}$ or -1, and how careful the candidate was in basic algebra (dealing with the square of ax or $\frac{1}{4}$, and the multiplication of expansions). It was noted that some candidates added their expansions. It was expected that the final answer should not be given as a set of 9 terms but that those concerned with x^2 would be clearly identified.

7) Despite the fact that 'Separating the variables' is a prominent part of the specification of this paper, a surprising number attempted to solve the differential equation without doing this. Most of those who separated the variables went on to score full marks, very few forgetting the constant of integration.

8) Quite a few candidates decided to produce the Cartesian equation before working on the individual parts; presumably some feel less confident when using parameters and prefer to get back to 'normal' ideas. It is a pity as this section of the syllabus can present fresh insights into their mathematical learning.

(i) A simple first part was most easily done by substituting the parametric equations into $y = 3x$, thus producing $t^2 = 4$ and the two relevant values of t. Those solving the Cartesian equation simultaneously with $y = 3x$ were not awarded any marks until values of t were found.

(ii) Most are well acquainted with the method of finding $\frac{dy}{dx}$ from parametric forms. A few thought the derivative of $\dfrac{1}{t+1}$ was $\ln(t+1)$ and others found reciprocating $-(t+1)^{-2}$ too awkward, but more errors crept in when substituting $t = -2$ and finding the gradient of the normal.

(iii) There were a number of instances where $y = 3x$ was used as the equation of the curve, but most took their normal from part (ii) and substituted the parametric equations into it.

(iv) This was well done but candidates must ensure that a result found earlier (say in part (i)) is also shown at the relevant stage.

9) (i) This was well done.

(ii)(a) Although the vast majority realised that $\int_1^e \pi(\ln x)^2 \mathrm{d}x$ was required, there were many cases of candidates changing $\int(\ln x)^2 \mathrm{d}x$ to $\int 2\ln x \,\mathrm{d}x$ or $\left(\int \ln x \mathrm{d}x\right)^2$. Those using 'integration by parts' were in the minority but generally prospered, except for sign errors at the second stage.

(ii)(b) The 'explanation' element was done poorly with very few able to show adequately the process leading to the given answer. Basic sign and integration errors prevented most candidates from scoring the very last mark on the paper.

4725 Further Pure Mathematics 1

General Comments

This paper proved to be accessible to the majority of the candidates. Most were able to demonstrate a good grasp of a wide range of specification topics, with most candidates producing completely correct solutions to a number of questions. Completely correct solutions to all questions were seen and there was no evidence of candidates being under time pressure. Very few candidates wrote their answer to one part of a question in the answer space for another question. Most candidates had sufficient space in the printed answer booklet and fewer additional sheets were required than in January.

Comments on Individual Questions

1) (i) Most candidates produced a correct solution, with minor arithmetic errors being the most common reason for loss of marks. Some candidates thought that the identity matrix was $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(ii) Most attempted the multiplication in the correct order and knew how to do it. Again minor arithmetic errors were seen. There was a small number of candidates who simply multiplied corresponding elements of the two matrices.

2) The induction step, to add the next term of the series, was understood by the majority of candidates, who then showed sufficient working to derive the correct form. The most common errors were to add $k + 1$, or $\frac{k+1}{k+2}$ to the given sum. A significant number did not produce a satisfactory statement of the induction conclusion.

3) The majority used the determinant of the coefficients correctly. There was some confusion on the candidates' part as to whether the determinant was zero, not equal to zero, or greater than zero for the equations to not have a unique solution. The most common error was to omit the negative value for k.

4) A significant number did not spot that the upper value of the sum was $2n$, although a number did work with n as the upper value and then correctly changed n to $2n$ at a later stage of the working. A fully factorised answer was not always produced, with candidates stopping at $n(8n^2 + 6n)$ or $n^2(8n + 6)$.

5) (i) Most candidates found the correct values for the modulus and argument.

(ii) The majority knew that the locus for (i) was a circle and for (ii) was a straight line. The centre of the circle was often in the wrong quadrant and many did not see that the circle passes through the origin and crosses the y-axis again. Most sketched a half line from the centre of their circle, but a significant number thought that $\pi/2$ meant that the line had a positive slope rather than an infinite one. A few candidates thought that the locus for (i) was a perpendicular bisector.

6) The methods required for this question were generally well known and applied successfully. The determinant and adjoint can be found in either order and both were done correctly by the majority, with most candidates dividing their adjoint by their determinant. Minor errors in evaluation of the cofactors was the most common reason for loss of marks.

7) (i) Most showed sufficient working to obtain the given answer. A number of candidates clearly wrote the numerator of the left hand side as $r+1-r-1$ and did not spot that this is zero.

(ii) The method of differences was well understood by most candidates, but some used the formula for $\sum r^2$ in the denominator. Some started the series at $r = 1$ and were not put off by the value $\frac{1}{0}$ from the first term. The cancelling was generally done correctly, but some made errors by not showing the differences for $r = n-1$. Some obtained a correct answer, but then did not combine the two fractions correctly, which meant that (iii) often went wrong.

(iii) This was generally well answered, but some gave the answer as a decimal, rather than giving it in fractional form.

8) (i) Quite frequently there was no indication of the coordinates of the image points, either on the diagram or in the working space for the question. The most common error was having the images of A and C still on their original axes.

(ii) The two transformations were correctly described by the majority of candidates and only minor slips in some elements of the matrices were seen. Very few checked that their matrices did multiply to the given matrix, which could have helped those who had made an error to try to find it.

9) (i) Most wrote down the conjugate root.

(ii) The sum and product of the roots was used successfully by most candidates, the most common error being to omit the negative sign in the value of a.

(iii) This part proved to be quite challenging. A significant minority solved a quadratic equation to obtain 16 ± 30i, and stopped. Many others realised that the square root of 16 + 30i was required, but simply gave the answer as 4 + √30 i or something similar. Some wrote down the correct square roots, with no working, so they had not used an algebraic method as required by the question. Many found two correct answers, but then did not realise that four roots were required. Those who realised that the equation had four roots often repeated the working for their first roots, rather than just writing down the conjugate roots.

10) (i) The algebraic techniques for rearranging the equation were rather poorly shown. Most could substitute, but then squared each term or rearranged to give a form that when squared still contained fractional powers.

(ii) Many did not understand that the roots of the cubic found in (i) are $\frac{1}{\alpha^2}$ etc. and so the required values may be written down from the coefficients of the cubic found in (i).

4726 Further Pure Mathematics 2

General Comments

The paper seems to have been a little more challenging this year.

Candidates should be encouraged to pay more attention to precise mathematical notation.

For instance:

- In question 1 even the best candidates seemed to be satisfied to leave their answer as a fraction with fractions in the numerator.
- In the questions involving hyperbolic functions the "h" was often missing – tanh$x$ being written as tan$x$, for instance.
- A very large proportion of candidates failed to include the "dx" with their integral notation.

Comments on Individual Questions

1) This question served as a good source of 4 marks at the beginning of the paper, but as mentioned above, a large proportion of candidates failed to write their final answer properly (and a few did not write it at all, merely giving values to their A, B and C). Only a very few lost all marks by not starting with the correct partial fractions.

2) (i) This was usually done well. Some candidates failed to divide correctly while others completed the division correctly but failed to write the equation of the line.

(ii) Most candidates took the correct route to proving this but failed to provide complete justification for the last mark.

3) (i) No difficulties were encountered in this part.

(ii) A number of candidates failed to answer the question properly and some who did failed to write their answers to the correct number of decimal places.

(iii) Some diagrams were very poor, including sketches of the "staircase".

4) (i) This question was not done well. The question relates only to that part of the curve that lies within the first quadrant, but many candidates did not realise this and so did not relate the conditions imposed by the question to the range of values of θ .

(ii) Many candidates asserted correctly that $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$, etc. and completed this part satisfactorily. Others seemed unaware of this and tried to use the compound angle formulae, often with little success.

(iii) Manipulation of surds to give a correctly written answer evaded many.

(iv) As with the previous question, only a few candidates sketched the curve well.

5) (i) In addition to the differentiation process it was necessary to justify why the positive value was taken; this was often missed.

(ii) The better candidates completed this successfully. Others had difficulty in differentiating a quotient and dealing with the powers. One or two found a series expansion by means other than using the Maclaurin series.

(iii) If the first parts were completed successfully then this part was usually also correct.

6 (i) Many candidates completed this question well. Practically every candidate started the process of integration by parts correctly. Some slipped up in the algebra or the substitution of the limits. When something goes wrong, many candidates "fudge" the resulting work to obtain the correct answer.

(ii) The biggest error here was to write $I_0 = -\frac{2}{5}$.

7) (i) Sketches were often poor. The value of the gradient being 1 when $x = 0$ was not used to inform the candidates of the nature of the curves at the origin. As a result many drew overlapping curves.

(ii) There was no problem with this standard integral.

(iii) There were at least three "otherwise" methods of finding this result. Once again, the fact that the answer was given tempted candidates into "fudging" their solutions rather than questioning their working if it does not seem to be coming out correctly. As a result of calculating this integral from scratch, a lot of work was often done. The inclusion of part (ii) to help in this part was missed by most.

8) (i) This part was done well, though many made a lot of work for themselves or went wrong by writing $2\sinh x\cosh x = \sinh 2x$ prematurely. The substitution back to x caused some problems.

(ii) The correct answer was found by most of those who got the integral correct in part (i), though it was disappointing to see many leave their answer with $\sqrt{12}$ rather than $2\sqrt{3}$.

(iii) The fact that the answer to part (ii) was finite was completely missed by practically all candidates. A typical answer was "infinite because the line $x = 1$ is an asymptote" ignoring the fact that it was also an asymptote to the curve when the area was being calculated.

4727 Further Pure Mathematics 3

General Comments

Overall this paper was found to be more demanding than those set in recent sessions, with some parts of questions being answered correctly by only a relatively small number of candidates. Nevertheless there were some excellent answers from the better candidates. For those in the middle range of ability there were several questions which did not present much difficulty, and these were well answered. There was some evidence of candidates not being able to use their knowledge of techniques from earlier units, including basic algebra; this occurred particularly in Questions 2, 3 and 8 as detailed below. There did not appear to be any problem with the length of the paper, and virtually all candidates reached the end. The number of exceptionally well presented scripts was smaller this time, and there were papers, across all ability ranges, which were so poorly written as to be indecipherable in places, with working scattered over the page and letters and figures which could be interpreted as almost anything. Some candidates have very small writing and it is not helpful to examiners when, for example, a complicated algebraic fraction is written within the space of one line of the answer paper.

Comments on Individual Questions

1) (i) This first part was done well, with most candidates scoring 3 or 4 marks. The standard method was known and applied correctly, but it was very common to see the complement of the required angle being given. Only a very small number started with the vector product instead of the scalar product; this earned credit only if the correct next stage of finding the modulus of the product was attempted.

(ii) Many candidates seemed unaware that the appropriate formula was in the *List of Formulae* supplied. Even when it was used, 40 sometimes had the wrong sign or was omitted. All other methods were longer, although substituting the point (1, 6, –3) into the equation of the plane and finding the plane parallel to p was quite common and straightforward. Answers which found the parameter of the point on the normal where it meets the plane were seen fairly often, although most attempts then used the coordinates of the point of intersection and the distance formula, rather than simply noting that the distance was 4 times the length of the unit normal.

2) (i) The manipulation of complex numbers required in this part was found to be much more difficult than had been expected. The easiest method, starting by extracting a factor of $e^{\frac{1}{2}i\theta}$ from both top and bottom, was seen reasonably often and the subsequent working was usually carried out correctly, although some fiddling of signs, 2 and i was sometimes evident. The most popular approach was to multiply top and bottom by the complex conjugate of $1-e^{i\theta}$, but most attempts then stopped at $\frac{i\sin\theta}{1-\cos\theta}$, candidates apparently being unaware that double angle formulae could then be used to lead to the given answer. Some variations on this method were seen, some of which could be made to work, but credit was given only if sufficient progress was made.

(ii) The concept of loci in an Argand diagram is within the specification for Further Pure Mathematics 1, but the majority of candidates scored only the first mark, for a circle centred on the origin as the locus of z. The second mark was for the correct radius and an indication of the anticlockwise sense but both of these were not often given. Strictly, the point 1 should have been omitted from the locus, but this was not realised by the candidates. The correct locus of w was seldom seen; it should not have been difficult to realise that w was a pure imaginary number, but candidates seemed not to appreciate this.

3) (i) Most answers gave the correct auxiliary equation, but rather fewer deduced the correct complementary function. The most common wrong answer was the function $(A + Bx)e^{-4x}$, which also meant that the follow-through mark at the end of part (ii) was not earned as there are two arbitrary constants when there should be only one. It is less usual to use this method of solution for a first order equation than for a second order one, but this equation is solved much more easily by this method than by other methods. Those who chose to use another method of solution in part (ii) often failed to state the complementary function here. A few candidates mixed up the terms "complementary function" and "particular integral", although they clearly knew how to solve the equation.

(ii) The majority of candidates used the correct form for the P.I. and usually solved the resulting equation correctly to give the values of the two constants. The final mark was for adding the C.F. and P.I. together, provided that they were of suitable forms. A fair number of candidates decided to use an "otherwise" method, which meant that they had to use an integrating factor. In this case the resulting integral $\int 5e^{4x} \cos 3x \, dx$ is not an easy exercise. Many attempted integration by parts but only the best candidates successfully followed the two-stage process through correctly. A minority opted for the $C + iS$ method, which was easier, and some very neat correct answers were seen.

(iii) In nearly all answers only the first mark was earned, by indicating that the Ae^{-4x} part of the solution approached zero for large values of x. Many candidates then answered the wrong question by stating that y was approximately equal to the P.I. in its trigonometrical form. Those who did give a range of values usually derived it incorrectly from their $p\cos 3x + q\sin 3x$ as, for example, $\pm(p + q)$ or $\pm$(larger of p and q); just a few correctly used $\pm\sqrt{p^2 + q^2}$, which was surprising as the form $R\sin(3x + \alpha)$ is within the specification for Core Mathematics 3.

4) All parts of this question were done well and nearly half the candidates gained full marks.

(i) Most answers showed the necessary stages in the proof, either with or without details of the associative property being shown. Some started from what was to be proved and then changed the left-hand side successively to arrive at $cba = cba$, which is not a satisfactory method. Solutions without the proper use of equal signs did not earn full credit. A few claimed that pairs such as a and bc were commutative, which happens to be correct but it would need to be proved from the properties given.

(ii) Almost all answers showed all seven subgroups correctly, even if they were not expressed properly as sets.

(iii) Although most found five correct subgroups, some could identify only three or four.

(iv) Most answers referred correctly, in some way, to the elements all having order 2, which implied that the subgroups were isomorphic. It was not clear whether candidates thought that the subgroups were those that they had found in part (iii) or all seven subgroups: probably few realised that there were more than five subgroups, but the marks were awarded without that detail being required. Some simply stated that the elements had the same order, without specifying order 2: this was not accepted.

5) (i) Most candidates had little difficulty in using the chain rule and changing the equation into one with variables u and x. In some cases the use of the chain rule was done in a roundabout way, but this was allowed. Details of the simplification into the required form varied between those who did it in one or two lines and those who divided by each term separately and told the examiners what they were doing: all such methods were accepted, but the former was preferred. There were, however, some who did not differentiate u^k correctly at the start or who made errors in their simplification. In a few scripts the writing was so poor that the letters x, y and u were almost indistinguishable from each other.

(ii) Most values for k were ± 1 or ± 3.

(iii) Those who had the correct value of k usually continued reasonably successfully towards the solution for y in terms of x. When mistakes occurred these were usually in carelessness with signs or in the final stage of changing back to y: some dreadful algebra of the form $u = x^2 + cx^3 \Rightarrow y = \frac{1}{x^2} + \frac{1}{cx^3}$ was occasionally seen. Others omitted the arbitrary constant in the integration. A follow-through mark was available, and usually earned, by those who had the wrong numerical value of k, but they were unable to make further progress because the resulting integral included the variables u and x. This did not appear to worry candidates as they simply treated u as a constant, rather than realising that something was wrong.

6) (a) Most answers scored three of the four marks. Nearly all checked the three properties of closure, identity and inverse correctly. Some gave the inverse element the form $-(ax+b)$ rather than $-ax-b$, but this was accepted. The mark which was not often gained was for stating that the expression $(a+c)x+(b+d)$ was a member of the set P or was of the form $px+q$: the majority said only that $a+c$ and $b+d$ were real numbers, which was not enough.

(b) Although most candidates showed some familiarity with modular arithmetic, not many realised that the set Q had 9 elements: the most usual answer was 3, followed by 6. For the inverse element $-2x-1$ was the most popular wrong answer. In part (iii) the allocation of four marks and the instructions to "find" and "hence determine" should have alerted candidates to the fact that more was required than the number 3 and "yes" or "no". It was quite common for the phrase "any element of Q" to be taken to mean "a particular element" and for the order of only, for example, $2x+1$ to be found; or else the orders of fewer than eight non-zero elements were found. For the award of both marks it was necessary to deal properly with the addition of $ax+b$ to itself until the identity 0 was reached under modulo 3 and to deduce order 3. The determination that the group was not cyclic required specific mention of the lack of an element of order 9, and this was obtainable only by those who had the correct order from part (i). Only the best candidates gained full marks for this question.

7) Throughout this question there was much confusion over notation. The letters P, Q and R referred to the vertices of the tetrahedron and the letters p, q and r to the lengths of the edges in the three mutually perpendicular directions. Many answers used these letters interchangeably for any purpose, including p, q and r being taken as vectors, with or without underlining. Mathematical notation is precise and when candidates choose to ignore what is clearly set out, they can be certain to run into trouble.

(i) Many diagrams would have benefitted from being larger and drawn with a ruler. A mark was given for a sketch which clearly showed at least one right angle at *O*, but this was generous for some of the sketches for which it was awarded. The confusion referred to above resulted in answers for the three areas being given as, for example, $\frac{1}{2}|P|\times|Q|$ or $\frac{1}{2}p\mathbf{i}\,q\mathbf{j}$: a method mark was awarded for some use of "$\frac{1}{2}$ base × height", but both marks were given only for correct simplified answers in terms of *p*, *q* and *r*.

(ii) Few scripts scored the single mark for this part. Many candidates used the formula for the area of a triangle, $\frac{1}{2}ab\sin\theta$, but when they applied it to the triangle *PQR* using the vector product most failed to identify θ as the angle between *RP* and *RQ*. There was quite a large number of candidates who did not use the definition of the vector product, as in the specification, but proceeded to evaluate it using the vectors $p\mathbf{i}$, $q\mathbf{j}$ and $r\mathbf{k}$: if this was done correctly, the marks were credited to part (iii) if the work was not repeated there.

(iii) This part was only answered properly by the highest-scoring candidates. It was necessary to give proper attention to the modulus signs, and statements such as $qr\,\mathbf{i}+pr\,\mathbf{j}+pq\,\mathbf{k}=\sqrt{(qr)^2+(pr)^2+(pq)^2}$ did not earn full credit. Many other candidates were confused by the notation and multiplied out expressions like $(p-r)\times(q-r)$ as if they were ordinary algebra. Other answers seemed to assume that *p*, *q* and *r* all had the value 1. Quite a number scored only the mark for expressing the sum of the areas of the three right-angled triangles correctly in terms of *p*, *q* and *r*.

8) (i) This was a standard request which was answered correctly by nearly all candidates. The very few who used a method other than de Moivre's theorem did not receive any credit.

(ii) Provided the result of part (i) was correct, this part was also done well.

(iii) Most candidates realised that they should obtain an equation by putting the polynomial in part (ii) equal to 1. Many then attempted to verify that $\cos\theta=\pm1$ was a solution; this scored a maximum of 1 mark, as it did not show that there were no other solutions, as required. Even this mark was not earned by those who tried only $\cos\theta=1$. Attempts at factorising the equation were quite common, but it was very worrying that 1 was often retained on the right hand side and statements such as $(2c^2-1)(8c^4-8c^2+1)=1\Rightarrow 2c^2-1=1$ were made. For those who factorised correctly the given result appeared quite easily, but full credit was given only if it was shown that the discriminant of the quadratic factor was negative.

(iv) Similar comments apply to this part as to part (iii), except that the factorising was more successful as there were no constant terms in the equation. Some of those who only verified that $\cos\theta=0$ was a solution did not earn the mark as no explanation at all was given: it was probably obvious to candidates, but when an answer is given, a clear reason must be given. Again, if the factorisation had been done correctly, the negative discriminant had to be clearly shown. Those who finished with $\cos^2\theta=0$ rather than $\cos\theta=0$ did not earn the final mark.

Chief Examiner's Report – Mechanics

General Comments

The use of answer books and online marking created few problems for candidates. However the scanning process emphasises any flecks and markings which have their origin in writing on the reverse side of a page. Most commonly this shows up as random decimal points or degree signs. For this reason the use of felt tip or gel pens is strongly discouraged.

Across all modules candidates were well prepared for the content of the question papers, with few indications that the syllabus had not been fully taught. The single factor which would raise performance is more careful reading of the questions. Whether this is horizontal/vertical (M1, Q3) or revolutions/radians (M4 Q1), it is a generalisation which applies across all levels of ability.

A second factor is the anticipation by candidates who expect that a problem will develop in a particular way. This is more evident in earlier modules. In M1, the acceleration found in 3(ii) was used in 3(iii) despite a significant change in the situation. In the same module candidates used a given acceleration to find the inclination of a smooth slope, such a request having been made in the January paper.

4728 Mechanics 1

General Comments

Many scripts indicated that candidates had prepared well for the examination. Conversely, a few suggested that entry for this module was premature.

If there was a weakness commonly seen in many answers it lay with insufficient thought being given to the situations modelled in the questions. More care in reading the questions, and picturing what was being described would have helped many candidates to improve their grade. The benefit of drawing a neat clear diagram in gaining a clear appreciation of the question remains insufficiently appreciated.

Comments on Individual Questions

1) Nearly all candidates gained full marks on this question, the only notable error being the (correct) calculation of the wrong angle.

2) Most candidates drew the correct diagram for the question, though a few had *P* moving on a horizontal table.

(i) A significant minority created an equation with only two terms either by imagining that *P* was stationary or that it had no weight. More solutions were marred by assuming that *P*, like *Q*, was accelerating downwards.

(ii) Candidates who obtained a wrong answer in (i) were able to gain a majority of the marks in (ii). The commonest error was to have inconsistent signs for the weight of *Q*, its acceleration, and the direction in which tension acted.

(iii) Nearly all candidates gained full marks in this part of the question.

(iv) A majority of candidates appreciated that, after *Q* struck the ground, *P* would continue to rise with a deceleration of 9.8 ms^{-2}. This further distance travelled was however often added to 0.36 m rather than 0.72 m. In a large minority of scripts the answer given was 0.72 m. This came about by assuming that *P* ceases rising immediately after *Q* struck the ground, or through assuming that *P* continued to rise with an acceleration of 0.98 ms^{-2}.

3) Very many candidates, perhaps a majority, answered a question in which the 6 N force made an angle of 60° with the *horizontal.* A special scheme for marking these answers ensured that otherwise correct solutions would lose only a single mark at the end of (ii).

(i) Diagrams here often showed an angle of 60° between the horizontal and the 6 N force. The variety of answers showed that many candidates did not understand which force was being demanded by the question.

(ii) Fortunately in many scripts candidates recalculated a value for "R" to be put in the formula $F = \mu R$. The correct answer 5.29 ms^{-2} and its misread counterpart 3.09 were seen with equal frequency.

(iii) It was expected that candidates would be likely to overlook the effect that removing the 6 N force would have on the magnitude of the frictional force. Such a mistake would likely lead to a score of 2/4. What was more surprising was that the loss of the tractive element of the 6 N force would be totally disregarded. However, very many solutions were seen in which the acceleration found in (ii) was used again in (iii), as the deceleration.

4 Full marks were common on this question. It was apparent that some candidates had done working out on the diagram in the question paper, so that full credit for erroneous final values could not be awarded.

(i) Nearly all candidates obtained full marks in this question.

(ii) Only a small minority of scripts contained any error, usually a result of misreading a value for v or t.

(iii) The correct strategy for finding the required time was used in nearly all answers, with arithmetic errors being the cause of most loss of marks.

5 The mathematics of momentum conservation were well understood. The most common error, seen in most papers, was to have particles travelling upwards on the plane at constant speed, while those moving down the plane accelerated. Thus Q would accelerate in (i), but move with constant speed in (ii). A smaller number of candidates lost marks through not interpreting their answers as "speed and direction" as requested.

(i) The belief that, at the moment of impact, P had speed 3 ms^{-1} lead to many candidates losing marks, and giving an answer of 1.32 ms^{-1} for its speed.

(ii) Candidates who had made the common error in (i) (referred to above) repeated the error with respect to the motion of Q in (ii), so that a speed of 0.22 ms^{-1} was commonly seen.

6 The question paper specified the way in which candidates should tackle the problem. Other approaches were possible and full credit was given to them.

(i) Very few candidates saw the relevance of the ring being smooth.

(ii) Candidates who drew large clear diagrams, with the aid of a ruler, were the most successful. It was clear that many who introduced sine or cosine of the angle $(90 - \theta)$ lacked the knowledge of how best to continue in (ii) or (iii). A few scripts showed answers which were not equations, but the most common error was $T\sin\theta = 7$, $T\cos\theta = 5$.

(iii) The process of solving the two equations led to much ingenious manipulation of the equations from (ii). Candidates who used techniques familiar from handling components in simpler statics problems were able to gain at least half marks, even if the values ultimately found were incorrect.

7 (i) There were very few solutions in which integration or constant acceleration formulae were used and nearly all candidates scored 4/4.

(ii) A majority of candidates obtained full marks, and the only error of note was a failure to demonstrate explicitly that $x = 0$ when $t = 1$, expected as the question effectively gave the anticipated value of x.

(iii) Most candidates were able to set up the quadratic equation for this part of the question. However many answers showed an inability to solve the correct equation and obtain the required values.

(iv) The request for a quadratic equation lead a small number of candidates to equate velocities. In the majority of papers candidates integrated the velocity of *Q* correctly and set it equal to the expression for the displacement of *P*. In many scripts the consequent 5 term cubic equation was legitimately reduced to the three term quadratic. The most awkward step for many candidates was the division by *t*.

The solution of the quadratic equation required a selection of the appropriate root and candidates often lost two marks by leaving 3 s and 6 s as their pair of answers.

4729 Mechanics 2

General Comments

The many good scripts seen demonstrated the high level of skill and mathematical understanding of the majority of candidates; only a small percentage of the candidature appeared to be totally unprepared for the demands of the paper. Candidates should pay attention to the precise requirements of questions and to the use of quality force diagrams with acceleration shown when appropriate.

Candidates should make use of the *List of Formulae* when calculating the centre of mass of uniform bodies.

Comments on Individual Questions

1) (i) Most candidates successfully found the change in the kinetic and potential energy of the sledge and its load. However candidates were less successful in combining the energies found, with some also including the work done by the resistive force.

(ii) This question was attempted in two distinct ways. The energy approach was more successful though some were unsure how to combine the energies correctly. Of those who used a Newton's Second Law approach, a significant number of candidates omitted the component of weight or even thought the acceleration of the sledge was zero.

2) (i) A well answered question by many candidates. Only a minority of candidates did not use power = force × velocity. Examiners saw an omission of g in the weight component quite often. A small minority of candidates thought the driving force was 21000N.

(ii) Although this part was well answered by the majority, a significant number of candidates had the engine providing a constant driving force rather than a constant power.

3) (i) It is common to be required to find the centre of mass of a composite body and most knew that moments needed to be used to do it. However, many wrong values were seen for the area of two simple shapes; for the square we were offered 68, 32 and 16, and for the triangle, $^{64}/_{3}$, 8 and 32, the latter proving particularly popular. Examiners also saw a number of errors for the centre of mass of the triangle even though this is contained in the *List of Formulae*. Those who did not know a correct method usually found the midpoint of the centres of mass of the square and triangle.

(ii) Candidates were not penalised for a wrong answer to part (i) and the majority gained full marks. Only a minority used the wrong trigonometric ratio in the calculation of the required angle.

4) (i)(a) This question proved difficult for most candidates. A minority considered the coefficient of restitution, or compared approach and separation speeds – these attempts were nearly all successful. The majority, who experimented with momentum and/or kinetic energy, were unsuccessful.

(i)(b) Normally answered very well, even by those who were unsuccessful in part (a).

(ii) This part proved to be accessible for the majority of candidates.

(iii) Some decided to bring in kinetic energy here, though most had a better idea of what to do. There were some who had confidently asserted that $0 < e < 1$ in part (i), but did not check their working when they arrived at $e = 2.17$ in part (ii), after making a sign error in the calculation of the velocity of B.

5) (i) Many candidates could find x and y in terms of t, but finding the equation of the trajectory proved more challenging. Some candidates quoted the equation of the trajectory and so ignored the 'hence' in the question. Others attempted to use $\frac{y}{x}$ or $x^2 + y^2$ in an attempt to eliminate t.

(ii) The majority of candidates found a quadratic equation in either x or t. However candidates are reminded that they should clearly show their method of solution of their quadratic equation so that they can be credited accordingly when arriving at wrong solutions from a correct equation.

(iii) Candidates who used $v^2 = u^2 + 2as$ were, on the whole, the more successful in this part. Often, those who found t first, used a rounded value in $v = u + at$, and therefore lost accuracy. A significant number of candidates did not realise that they needed to use velocity to find the direction of motion.

6) (i) There was much confusion in this question, which could have been addressed if candidates had a good quality force diagram which also included acceleration. Often examiners saw a wrong normal reaction of $0.3g \sin 30$ from resolving in the horizontal direction, which should have included a component of acceleration. The method of resolving vertically and horizontally was the most successful approach.

(ii) Those candidates who had made a good attempt in (i) were usually also then successful here. Some candidates did not like the idea of two contact forces, and consolidated them into one force acting at 15° to the horizontal or made them equal.

(iii) The reference to mechanical energy was to cover all the energy within the specification for this module. Many candidates only considered kinetic energy and omitted potential energy. These candidates were able to access the majority of the marks despite this omission.

7) (i) The centre of mass of the cone was sometimes placed ½, ¼ or ⅔ of the way from V to the midpoint of PQ. Most realised that they should take moments about V and had a good attempt though it was common to make a mistake with at least one of the distances. Those who attempted taking moments about other points or resolving were rarely successful.

(ii) This question proved to be the most difficult, with few correct solutions seen. The most successful approach was to take moments about P. It was common to see, incorrectly, the normal contact force to be equated to the weight. Often candidates were unclear as to where to place the normal contact force.

4730 Mechanics 3

General Comments

This paper was taken by many able and very well prepared candidates, who showed good knowledge and understanding of the content. However, the paper did prove to be a little more demanding than some in recent sessions. The paper was also taken by some candidates who struggled to show that they had any real understanding of any of the topics, and others who did one or two of the questions very well, but gained very few marks on the rest of the paper.

While the work of the majority of candidates was reasonably neat and well presented, some candidates might find they would benefit by explaining what they are doing – to both the examiner and themselves. For example, stating whether they were taking moments or resolving, about which point or in which direction, and for what part of the body, or for its whole, in Question 2, might have helped some candidates clarify their thought processes and produce a better answer. Similarly, candidates who made it clear if they were working with gravitational or elastic potential energy in Question 6, and if they were dealing with point X or point Y, were able to avoid the confusion that was apparent in some less well-organised answers.

Comments on Individual Questions

1) This question was answered completely correctly by most candidates. The requirement to show that $\cos\theta = 0.8$ needed either a triangular diagram showing momentum/impulse or an indication that the result followed from the conservation of momentum perpendicular to the direction of the impulse.

   A small minority of candidates made an error in the value of I, usually by omitting the mass of 0.3 kg, or by giving the value of I^2.

2) The quality of work on this question varied from concise efficient answers, using the right equation for each part to a few inaccurate jottings meant to be the resolution of forces or an attempt at taking moments. Apart from candidates who hardly got started, the vast majority realised that the trigonometric ratios needed were 0.6 and 0.8, and there were few who worked out approximate angles and then used them. Candidates should be encouraged to state what they are doing at each stage; they should make it clear if they are resolving or taking moments, whether it is for the whole body or for part of it, and state the direction (if resolving) or the point about which moments are being taken.

   (i) The best candidates took moments about C for the whole body. It was at least as common for candidates to take moments about A for each rod, and also to resolve vertically for the whole body. Other successful, though lengthy, approaches involved introducing the tension in the string and/or the forces at A at this stage. Some candidates incorrectly introduced friction forces at B and C. A minority of candidates abandoned finding the 374 N force, but gained a mark for assuming it and using it to find the vertical force at C.

   (ii) Those who succeeded in part (i) had little trouble here – some of those using longer methods had already worked out the tension on the way to the answer to part (i).

   (iii) Almost all realised that the horizontal component followed through from their answer to part (ii), and that only a little work was needed to find the vertical component. Most candidates gave the directions clearly and correctly.

3) (i) Almost all candidates correctly used the d*v*/d*t* form for acceleration in their differential equation; some forgot the minus sign and some others the mass. Most integrated to give an equation involving t and $1/v$, though there was some confusion about working out the value of the arbitrary constant. Any correct form of the expression giving v in terms of t was accepted; candidates who did not give an explicit expression lost a mark.

(ii) Almost all candidates gained the mark for attempting to find t when $v = 0.2$, although some then went into completely the wrong approach using, for example, constant acceleration formulae. The distance travelled in this time could either be found by integrating the expression found for v in part (i) or by returning to the equation found at the start of part (i) and using the $v\mathrm{d}v/\mathrm{d}x$ form for the acceleration. There were many good answers using both methods, and also many where mistakes crept in.

4) (i) Most candidates did this part very well, though a few lost a mark by making a sign error, or forgetting part of a term. The question asked for candidates to show the result, so it was not enough just to quote it. Some candidates did not make it clear that the approximation $\sin\theta \approx \theta$ applies only for small angles.

(ii) Most candidates correctly used the correct expression for angular displacement to work out the first value of t, and many of those also found the second value. Some candidates tried, invariably without success, to work with the expression for SHM from part (i) or even from the expression originally derived.

(iii) Again, most candidates used the correct expression for angular velocity, though some preferred to use the formula $v^2 = \omega^2 (a^2 - x^2)$. This sometimes led to confusion between the angular speed of P and its linear velocity.

5) (i) This was usually done correctly, though not always efficiently.

(ii) Most candidates did this correctly and efficiently, though some made sign errors and a small minority used components perpendicular to the line of centres.

(iii) Again, most candidates knew how to do this. Follow through credit was given for those who had made an error in part (ii). Some further sign errors were seen, and some candidates worked out the reciprocal of the required answer.

(iv) This part caused considerable difficulty, with the most common answer being 3.61 ($= \sqrt{13}$). Candidates were expected to realise that v_A is equal to the component of A's velocity perpendicular to the line of centres before impact, and that this could be worked out from knowing A's velocity parallel to the line of centres before impact and the given value of $6.5m$ J for the kinetic energy.

6) (i) Showing the increase in the gravitational potential energy was done very well, though some candidates made it more difficult than it needed to be by choosing an awkward zero level to work from. A few candidates wrongly indicated an increase rather than a decrease. While there were very many excellent answers for the elastic potential energy, mostly set out very clearly, there were also a considerable number where candidates made errors in the calculation, and some where they thought the 'extension' to be used in the formula was the difference between the length of the string when P was at X and at Y.

(ii) The verification was sometimes omitted, and candidates who had the wrong idea of 'extension' in part (i) often made little progress. Most attempted to use the correct method to find the maximum speed of P, though some made a sign error, and others used their own original versions of the energy increase and decrease, rather than the expressions quoted in part (i). This made finding the speed difficult (if their expressions were correct) or impossible (if they were not).

7) (i) This piece of bookwork was done very well by most candidates. Some produced wrong answers for v^2 because they omitted the initial kinetic energy, others because they did not include both parts for the change in potential energy. The main mistake in finding the tension was to omit the weight component from the equation, or to use $mg\sin\theta$.

(ii) Most realised exactly what had to be done here, and there were many fully correct answers.

(iii) The most efficient solution relies on candidates realising that the velocity at the highest point is the same as the horizontal component of the velocity when the string becomes slack, and then using conservation of energy for the point of projection and the highest point. Many candidates did not realise that the speed could be found this way, and tried to use the vertical component of the speed at the point the string becomes slack to find the greatest height. Although some did this correctly, many forgot that the height required was from the initial point of projection.

4731 Mechanics 4

General Comments

The work on this paper was generally of a very high standard, with half the candidates scoring more than 60 marks (out of 72). The great majority of candidates showed a good working knowledge of most of the topics being examined, and had sufficient time to produce substantial attempts at all the questions. Most presented their work clearly.

Comments on Individual Questions

1) This question, on constant angular deceleration, was answered correctly by about three-quarters of the candidates. The only common error was the use of 10 radians in part (iii) instead of 10 revolutions.

2) Finding the centre of mass of a rod of variable density was very well understood, and about 70% of candidates scored full marks on this question. There were quite a few errors in the integration by parts, such as omission of a factor *a*.

3) This question, on rotation and energy, was answered correctly by about half the candidates. Part (i) requires work done by the couple and potential energy, but many regarded it as a position of equilibrium rather than one of instantaneous rest. Finding the angular acceleration in part (ii)(a) was done well, although the moment of inertia was sometimes calculated incorrectly. In part (ii)(b) the couple was quite often omitted from the equation of motion.

4) About 60% of candidates scored full marks on this question on the energy approach to equilibrium. Some candidates did not realise that the extensions of the strings were given by the lengths of *AD* and *BD*, but the great majority wrote down a correct expression for the total potential energy. Many then lost marks by not obtaining the given expression in part (i) convincingly. Using this expression to determine the position of equilibrium and its stability in parts (ii) and (iii) was done very well indeed.

5) In part (i) the derivation of the moment of inertia of the sphere was well understood and was very often completed accurately, although omission of the factor ½ and algebraic slips spoilt many attempts. Some based their calculation on the integral of $x^2 y^2$ instead of y^4. In part (ii), the period of a compound pendulum was handled well, although some of those who chose to use the standard formula made no attempt to find the centre of mass. About 40% of candidates scored full marks on this question.

6) Most candidates were able to obtain the given relative speed and direction in part (i) and use it to find the shortest distance in part (ii). Parts (iii) and (iv) were almost always attempted by drawing a velocity triangle, although this sometimes corresponded to the wrong relative velocity (*P* relative to *Q* instead of *Q* relative to *P*) or to the velocity of one of the ships being opposite to its actual direction. Some had a correct diagram and calculated the angle correctly but could not convert this to the correct bearing. About one third of the candidates scored full marks on this question.

7) Most candidates found the moment of inertia of the block correctly in part (i), and also obtained the angular acceleration and angular speed correctly in parts (ii) and (iii). When finding the force acting at the axis in part (iv), those who considered horizontal and vertical components were often able to obtain the given result efficiently, and about a third of the candidates scored full marks on this question. However, very many considered components parallel to *BA* and *AD*, treating these as the radial and transverse directions and therefore having incorrect expressions for the accelerations.

Chief Examiner's Report – Statistics

General Comments

The number of candidates for Statistics units has increased substantially over the past few years. The standard of good candidates is generally very high, although there is evidence of "teaching to the test" and candidates are often unable to demonstrate understanding as opposed to the ability to carry out familiar procedures mechanically.

Reports by Principal Examiners continue to emphasise the same points from year to year. It is plain that many Centres have taken careful account of the information and advice given in these reports; it is equally clear that others have not. Centres benefit from careful study of these reports and Examinations Officers and Heads of Department are strongly urged to ensure that the reports are disseminated among all those who teach the relevant units.

4732 Probability and Statistics 1

General Comments

Many candidates showed a reasonable understanding of a good proportion of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. There were several questions that required an interpretation to be given in words, and these were often answered poorly.

A significant number of candidates lost marks by premature rounding (especially in 4(iii), 5(iii) and 8(iv)) or by giving their answer to fewer than three significant figures without having previously given a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or the intermediate answer correct to several more significant figures.

A significant number of candidates lost marks through failing to read the question properly, particularly in questions 1(ii), 3(ii) and 8.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Candidates generally had no trouble using the spaces allocated for each part-question and very few answered questions in the wrong space. Some candidates ran out of space and continued on an extra sheet, but without any indication that the examiner needed to look at the extra sheet. Centres should note that if candidates run out of space for a particular answer, they should ask for extra sheets and they should indicate in the normal space for the particular question that there is work on additional sheets. These sheets should then be attached at the back of the answer book.

Use of statistical formulae and tables

The *List of Formulae*, MF1, was useful in questions 1(i)(a), 2, 3(i) (for binomial tables) and 5(iii). In question 1(i)(a) a few candidates quoted their own (usually incorrect) formulae for r, rather than using one from MF1. Some thought that, eg, $S_{xy} = \Sigma xy$. Others used the less convenient version, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}}$ from MF1, but most of these completely misunderstood this formula, interpreting it as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{\sqrt{\{(\Sigma x-\bar{x})^2\}\{(\Sigma y-\bar{y})^2\}}}$.

In question 2, Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes miscopied as $\frac{6\times\Sigma d^2}{4(4^2-1)}$ or $\frac{1-6\times\Sigma d^2}{4(4^2-1)}$. Additionally, a good number of candidates found $\Sigma d^2 = 6$ correctly, but then squared this value before substituting.

In question 3(i), some candidates' use of the binomial tables showed that they thought the entries to be individual, rather than cumulative, probabilities. Many did not know how to use the tables to handle $P(X = 10)$. Responses to question 3(i) gave evidence that many students (understandably!) prefer to use the binomial formula rather than the tables. There was no problem with this in this case, but centres should be aware that questions are sometimes asked in which the use of the formula is laborious whereas the use of the tables is quick.

In question 4(iii) candidates needed to use formulae for the mean and standard deviation that are not given in MF1. Some had clearly learnt the formulae by rote but did not understand them. For example class widths were often used instead of mid-points.

It is worth noting again that candidates would benefit from as much guidance as possible on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates used exclusively the versions given in MF1 (except in the case of *b*, the regression coefficient, not tested in this paper). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) Many candidates failed to appreciate the significance of the fact that the value of *r* is close to zero.

(i)(a) Most candidates calculated *r* correctly, but they did not give an answer to more than three significant figures in order to show that they had done the calculation rather than copying the answer from the question. These candidates lost a mark. A few made errors such as those mentioned above.

(i)(b) Many candidates failed to give an answer in context and so lost the mark (eg "Little correlation between the two factors"). Others gave context but stated that the correlation was negative and so income goes up as distance goes down or vice versa, thus missing the point entirely.

(i)(c) Many candidates thought that the value of *r* would increase or become closer to 0.

(ii) A very common error was to state that the estimate would be reliable because the given value was close to the mean and therefore was probably within the range of the original data. Others stated that it was unreliable because it was <u>not</u> within the range of the original data or because this value might be an anomaly. None of these answers gained any marks. Serious misunderstanding was evident in answers such as: "Not reliable as we don't know the distance".

A few candidates failed to read the question and attempted to calculate the equation of the regression line.

2) Some candidates appeared to be unfamiliar with questions on rank correlation where the data is given in this format. They had difficulty obtaining the ranks and many resorted to using alphabetical order. Amongst those who did find ranks, the usual errors in using the formula (as described above) were not uncommon. Some candidates did not write down their ranks, but began with their *d* values, which made their solution unclear and gave more opportunity for errors.

3) (i)(a) Amongst those who used the table for this part, there was some confusion about whether to find $1 - P(X \le 10)$, $1 - P(X \le 9)$ or $1 - P(X \le 11)$. Some just looked up "10" in the table and omitted to work out "1 –". Some who used the formula found $P(X = 10)$ or $P(X = 9)$ or $1 - P(X = 10)$. A few candidates used the formula to evaluate $1 - P(X \le 10)$ thereby wasting a considerable amount of time and often making errors. A few candidates confused this question with one on the geometric distribution, finding, for example, 0.15^{10} or $1 - 0.85^{10}$.

(i)(b) Some candidates just looked up "10" in the table. Others found $1 - P(X \le 9)$. A few used the formula but omitted the coefficient, or used powers 9 and 3 instead of 10 and 2.

(i)(c) Almost all candidates answered this part correctly. Some used the distribution given in part (ii). A few used the formula for the variance of the geometric distribution. A small minority used methods such as $\mu = np = 12\times0.85 = 10.2$; $\sigma^2 = \Sigma x^2 p - \mu^2 = 12^2\times0.85 - 10.2^2 = 18.36$.

(ii) Many candidates found either $\frac{1}{4}\times\frac{3}{4}$ or $(\frac{3}{4})^2$ but not both. Thus a common response was $\frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4}$ which appears to confuse finding $P(Y = 1)$ in two ways with finding two values of Y. A few omitted to multiply $\frac{1}{4}\times\frac{3}{4}$ by 2. Some of those who found both $\frac{1}{4}\times\frac{3}{4}$ and $(\frac{3}{4})^2$ then multiplied them without multiplying by a further 2 for the two possible orders: 0, 1 and 1, 0. Others added them together. A large number found three products: $(\frac{3}{4})^2$, $2\times\frac{1}{4}\times\frac{3}{4}$, and $(\frac{1}{4})^2$ but were unsure of which of these were relevant and what to do with them. Some candidates found $2\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}$, presumably thinking that P(The 1st value of Y is 1) is ${}^2C_1 \times \frac{1}{4}$. A few found $(\frac{1}{4})^2$ and $2\times\frac{3}{4}\times\frac{1}{4}$. Some of these then proceeded to find $P(Y_1 + Y_2 = 2)$, whether from a misread or a misunderstanding it was impossible to tell. Many candidates wrote expressions such as ${}^2C_x \times 0.25^x \times 0.75^{2-x}$ without assigning any value to x.

4) (i) Many candidates correctly verified that the height of the 31-35 block was equal to its frequency density and hence the height of the 28-30 block is equal to its frequency density ($28\div5 = 5.6$), so $4 \div 3 = 1\frac{1}{3}$. Others just found $4 \div 3 = 1\frac{1}{3}$ without distinguishing between height and frequency density. Sadly, many of these believed that $4 \div 3 = 1.3$. Many calculated the class widths incorrectly as 2 and 4 instead of 3 and 5. A few of these continued with an otherwise correct method: $5.6 \div 28 \times 4 \times 4 \div 2 = 1.6$. Some found frequency/class width = 4/2 = 2. Many candidates just used the ratio of the heights without considering the different class widths.

(ii) Most candidates stated that the median was the 25th or 26th value or the mean of these two. Some proceeded to calculate a precise estimate of the median, whereas others just stated that the median is the 21st or 22nd in the 31-35 class. The calculation was not essential in order to gain full marks. A few concentrated on the value of 33 rather than the median, explaining either that it is the 14th value in the 31-35 class and therefore below the median (the 21st or 22nd value) or that it is the 18th value overall and that therefore there are more values above 33 than below. Any of these methods could score full marks as long as they were clearly expressed.

Some candidates stated that the median was in the upper half of the 31-35 class, without justification. Some candidates stated that the median lies within this class, but without any indication of where in that class the median lies. These often went on to say that therefore the median is equal to 33, as this is the mid-point of that class. A few gave the correct answer for an inadequate reason such as that there are more people in the 36-45 class than in the 28-30 class.

Answers involving skew were seen, but scored no marks. For example: "The median is less than 33, as the values are positively skewed".

A few candidates ignored the total frequency of 50 given in the question and calculated their own, sometimes arriving at 60. These candidates were then looking for median in the wrong place.

(iii) This question was found difficult by many candidates. Some used incorrect midpoints, for example 40 or 41 for the 36-45 class or 52.5 for the 46-60 class. Some candidates found $\Sigma(fx)^2$ or Σx^2. Many used class widths as their values for x. Many candidates succeeded in the GCSE-level task of finding the mean but had no idea at all how to find the standard deviation. Some omitted to subtract $\bar{x}^2$. Others found $\sqrt{\frac{fx^2}{\Sigma fx} - \bar{x}^2}$ or $\sqrt{\frac{\Sigma fx^2 - \bar{x}^2}{\Sigma f}}$ or $\sqrt{fx^2 - \bar{x}^2}$. As usual, $\frac{\Sigma fx}{50} \div 4$ and/or $\frac{\Sigma fx^2}{50} \div 4$ were seen. Most of those who attempted to use $\sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}}$ made errors. Some omitted the values of "f"; others found $[\Sigma(x - \bar{x})]^2$. Major errors such as $\frac{\Sigma x}{4}$ and $\frac{\Sigma x^2}{4}$ were not uncommon. A few candidates lost the final accuracy mark because they used their 3-significant-figure value for the mean in their calculation of the standard deviation.

(iv) Most candidates answered the majority of these parts correctly. The parts most frequently answered incorrectly were parts (b) and (d).

5) (i) Most candidates answered this simple GCSE-level question correctly. Some gave denominators of 8 and 7 instead of 9 and 8. A few answered the question "with replacement".

(ii) This part was well answered by many candidates.

(iii) Some candidates omitted to subtract μ^2. A significant number of candidates found $\Sigma(xp)^2$ or Σxp^2. As is usual, a few candidates divided Σxp and/or $\Sigma x^2 p$ by 3. A few lost the final accuracy mark because they used their 3 significant figure value for E(X) in their calculation of Var(X). Some attempted to use the formulae for the mean and/or variance of a binomial or geometric distribution.

6) In parts (i)(b), (ii)(a) and (ii)(b) some candidates found the relevant number of arrangements but did not go on to find the probability.

6) (i)(a) Almost all candidates answered this part correctly.

(i)(b) Many candidates did not recognise the key fact that there are six items to be dealt with. These candidates often considered the five others by themselves, using 5! rather than 6!. A few tried to use $^7P_2$. Many others did recognise the six items, but omitted to arrange Tom and Jerry. These correctly found 6! but failed to multiply by 2.

(ii)(a) Most candidates saw the need to consider the four boys and the three girls separately, but many did not know how to do this. Some common attempts were these: $3! \times 4! \times 2$, $3! + 4!$ $\frac{3!\times4!\times5}{7!}$, $\frac{3\times4!}{7!}$, $\frac{7!}{4!}$, $\frac{7!}{3!\times4!} \div 7!$, $\frac{1}{7!}$, $\frac{3!}{7!}$, $\frac{^7P_3}{7!}$, $\frac{^7C_3\times{}^7C_4}{7!}$.

(ii)(b) Many candidates did not recognise the key fact that there are five items to be dealt with. Many others did recognise this but failed to see the need to arrange these five items or to arrange the three girls. Some common attempts were these: $\frac{5!}{7!}$, $\frac{5!\times3}{7!}$, $\frac{5\times3!}{7!}$, $\frac{5}{7}$, $\frac{5!}{7}$, $\frac{^5C_3\times{}^5C_4}{7!}$ and $\frac{4!\times3!}{7!}$.

7) (i) A large minority of candidates thought that, in a "y on x" regression line, the independent variable is y.

(ii) Many creative and interesting diagrams were seen, purporting to show which actual "squares" are referred to by the phrase "least squares". A few candidates drew the correct lines on the diagram but failed to give a full description of the meaning of "least squares", usually omitting either "sum" or "squares". Some showed horizontal, instead of vertical, distances on the diagram. Others showed the shortest distances from the points to the line. A large number of candidates appeared to have little idea about least squares regression lines.

(iii) Many candidates did not appreciate the difference between rank correlation and linear correlation and answers such as "Negative" or "Near –1" or "About –0.5" were common. Those who correctly gave "–1" sometimes found it difficult to explain why. Inadequate reasons such as "Strong negative correlation" or "The line slopes down" or "The points are about the same distance away from the line on both sides" were commonly seen. A few candidates thought that, because the correlation is negative, $r_s = 0$. Others thought that r_s is positive because the correlation is strong. A few candidates tried to estimate the distances and calculate a value for r_s. Others thought that $r_s = 1$ because "the ranks will be the same".

(iv) Just "Negative" was sufficient to gain the mark and many candidates did so. Some gave the better answer of "Close to –1". Some gave answers such as "Close to 1 because the points are close to the line". Answers showing total misunderstanding, were not uncommon, such as "Close to zero because the points are close to the line".

8) Many candidates used the geometric distribution, but with powers that showed that they had not read the question properly and were only considering one player rather than all four. These candidates usually gave the following responses:

(i) $\frac{1}{6}$ **(ii)** $\frac{5}{6}\times\frac{1}{6}$ **(iii)** $\left(\frac{5}{6}\right)^2$ or $\left(\frac{5}{6}\right)^2\times\frac{1}{6}$ **(iv)** $\left(\frac{5}{6}\right)^3$ or $\left(\frac{5}{6}\right)^2\times\frac{1}{6}$.

Others attempted to use combinations. A common misapprehension was that answers to questions on the geometric distribution necessarily involve $(1-p)^r \times p$.

8) (i) A common incorrect answer was $\frac{1}{6}$.

(ii) Common incorrect answers were $\frac{5}{6}\times\frac{1}{6}$ and $(\frac{5}{6})^8\times\frac{1}{6}$.

(iii) Common incorrect answers were $(\frac{5}{6})^2$, $(\frac{5}{6})^9$, $1-(\frac{5}{6})^8$, and $(\frac{5}{6})^8\times\frac{1}{6}$.

(iv) The correct answer involves adding four terms:

$(\frac{5}{6})^9\times\frac{1}{6}+(\frac{5}{6})^{10}\times\frac{1}{6}+(\frac{5}{6})^{11}\times\frac{1}{6}+(\frac{5}{6})^{12}\times\frac{1}{6}$).

Many candidates gave just the first of these while others gave the first three. Some gave these four plus an extra term, either $(\frac{5}{6})^8\times\frac{1}{6}$ or $(\frac{5}{6})^{13}\times\frac{1}{6}$. Another common incorrect answer was: $(\frac{5}{6})^2\times\frac{1}{6}$. A few attempted the elegant method of $(\frac{5}{6})^9-(\frac{5}{6})^{13}$, but many gave incorrect attempts such as $(\frac{5}{6})^9-(1-(\frac{5}{6})^{12}$).

A small minority of candidates interchanged $\frac{5}{6}$ and $\frac{1}{6}$.

4733/01 Probability and Statistics 2

General Comments

There were many good candidates for this paper, but also many who had a very limited understanding of much of the specification. The spread of marks was unusually wide.

Some of the main points are:

- Random sampling must use random numbers (and not "hats", etc)
- Omission of $\sqrt{n}$ where required loses a lot of marks
- Probability density functions are still very poorly understood as a concept
- Hypothesis tests are often very muddled and with continuous variables, many candidates still confuse μ and $\bar{x}$, which is a serious blunder
- The Central Limit Theorem is widely misunderstood
- Modelling conditions for the Poisson distribution are not understood. There is too much reliance on lists in textbooks, many of which are misleading or wrong; see question 8(a)(ii).

An increasing number of candidates have calculators that provide probabilities or numerical integrals directly. Use of such calculators is permitted, but those who use them need to be aware that if they show insufficient working, a wrong answer cannot score any marks – even if it is "nearly" right. Candidates for this unit are expected to show full working at every step, particularly when carrying out standardisation using the normal distribution.

Comments on Individual Questions

1) All that is required here is to *number the houses* (or use the house numbers), and *select using random numbers*, *ignoring numbers outside the range* (and possibly repeats). As has been said in previous reports, the examination is testing knowledge of the specification, which includes random numbers – so responses such as "select numbers randomly", "put numbers into a hat" or "a lottery machine" are not correct. Some said "put the numbers into a random number generator", which suggests a lack of understanding of what random numbers are.

   It should be noted that picking a three-digit random number between 0.000 and 0.999 and multiplying by 263 is a *biased* method (1000 numbers cannot be equally distributed between 263 houses).

2) Those who started by using symmetry to find μ more often got the whole question right than those who wrote down two standardised equations. This latter is the "routine" way; as usual many did not use z-values, got signs wrong or made hard work of solving two simultaneous equations. Substitution should *not* be used in this context; elimination is always far easier. There is in fact no need to find σ explicitly as the ratio of 1.5 : 1.645 is all that is used.

3) Completely correct answers were pleasingly often seen. However, some candidates struggled to begin this question. Instead of finding the critical region, they often tried to standardise 15 or even 16 (which is n) with 20. Those who omitted the $\sqrt{16}$ factor lost a lot of marks.

4) (i) Many candidates did not notice the "0 (otherwise)" in the PDF and drew a parabola that extended well past 0 and 4.

   (ii) Most knew roughly what to do, but the pure mathematics required caused many candidates to struggle. Some tried to integrate x^2 and $(x - 2)^2$ separately; others could not multiply out $(x - 2)^2$ correctly, $x^2 - 2x + 4$ being common. The value of the mean could be found by symmetry and there was no need to integrate to find it.

(iii) Very few realised the fundamental misunderstanding in this statement. X is not an event that "occurs"; x represents a possible value of X and good answers focussed on the crucial missing word "values". However, this was too subtle for most candidates. Some candidates misunderstood and gave answers, such as "no because numbers close to the mean are more likely" or "no because numbers either side of the mean are equally likely" or "no because the variance gets bigger as you move away from 2". It was therefore decided to award the mark to anyone who said that they agreed with the statement and gave a reason that mentioned y or said more about the shape of the graph than "from the graph".

5) (i) A basic binomial hypothesis test. Those who used $P(R = 1)$, as opposed to $P(R \leq 1)$, were unable to score more than 3 out of 7. Too often hypotheses were stated in terms of μ rather than p. As usual, many conclusions lost a mark either by failing to give the answer in context ("the % <u>who book with travel agents</u> has been reduced") or who were too assertive, omitting the key words "there is evidence that…". As has happened previously, some candidates attempted to use the normal distribution.

(ii) Many realised that you could not deduce cause-and-effect or said that there would be other factors involved.

6) (i) A wholly routine hypothesis test question, albeit unstructured. There were many excellent solutions. The obvious mistakes were common: failure to state the hypotheses correctly, wrong calculation of the variance, failure to multiply by 50/49, and, most seriously, failure to use 24.3 rather than the sample mean of 26.28, as μ.

(ii) The key word in the question is "possible", so the only answer is "because n is large enough". "Unknown distribution" means that it is <u>necessary</u> to use the CLT. Again, there were incorrect answers such as: "You are using the sample mean", or "no as it is normally distributed already"; many candidates seem to think that all continuous random variables are (exactly) normally distributed regardless of any other considerations.

7) (i) Most got this right, or got 14 instead of 15. Some candidates looked for 0.146 in the left-hand tail instead of the right-hand tail. Many candidates have been taught to use $n - 1$ in this type of question; this seems a successful strategy.

(ii) Most recognised Po(44) and were able to go to N(44, 44). As usual there were subsequently problems with the continuity correction and/or $\sqrt{\lambda}$.

(iii) This is a common type of question, but some candidates were unable to appreciate that the underlying distribution was B(40, 0.146), and scored no marks. Some attempted to use the exact distribution, but as the question explicitly required an approximation, they scored no more marks. This applies too to those who used calculator software to get the "exact" answer.

8) (a)(i) Many gave the obvious answer that several people might call about the same fire.

(a)(ii) The poorest-answered question on the paper. As has been said before, the lists given in many textbooks are unhelpful or even wrong, but far too many candidates just reproduce these lists with little thought or understanding. "Singly" is part of "independently", if indeed it is required at all. "Randomly", or "randomly in time or space" (as often seen), is meaningless as an answer to the question. "Equal probability" is the binomial condition and therefore the wrong response here. The relevant condition is that calls have to occur *at constant average rate*. Some said "constant rate" and then unwittingly emphasised the importance of the omitted word "average" by saying something like "but fires don't occur at equally spaced intervals". Some knew what was necessary but referred to varying average rate over a year (e.g. bonfire night) rather than, as the question required, over a day.

(b)(i) Candidates needed to know how to use the Poisson formula, and many did so; those who tried to use tables scored no marks. Those who used a calculator, showed no working and omitted a term also scored no marks. A very common error was $P(> 2) = 1 - P(\leq 1)$.

(b)(ii) Most made a good attempt at $P(R = 0) \times P(S = 1) + P(R = 1) \times P(S = 0)$, but a surprisingly common mistake here was to use 0.4060 and 0.1991 from tables, which are the values of $P(R \leq 1)$ and $P(S \leq 1)$. Many, however, attempted to add up all four probabilities.

(b)(iii) Many did this algebra well, though there were many omissions of brackets, such as $e^{-\lambda + \mu}.\lambda + \mu$, which lost marks. A few failed to note the instructions and did the calculation with e^{-2} and e^{-3}.

4734 Probability and Statistics 3

General Comments

The paper was of a similar standard to that of June 2010 and many candidates scored very high marks. Candidates usually presented their work acceptably but with some it was not always easy to decode, particularly when words and numbers are over-scored. It is expected that a line should be put through work that is not required and the replacement then inserted.

Q1(iii) was the worst answered, many believing that the equality of mean and variance implies a Poisson distribution. It was pleasing to find that more candidates successfully answered Q5(ii) which required the CDF and PDF of Y where $Y=1/X^2$.

Comments on Individual Questions

1) (i) Most could obtain the value of E(S) correctly and a minority was able to find Var(S).

(ii) E(T)=Var(T) was usually shown correctly, but there was some fudging.

(iii) A few could answer this correctly, see above.

2) (i) The procedure for finding a confidence interval for a proportion seems to be very well known and many candidates scored full marks.

(ii) It was rare to find candidates who used the variance estimate in Part (i), and so most found themselves in difficulties.

(iii) It was hoped that candidates would refer to the normal approximation to p_s , but this was rare. Reference to the variance being an estimate was acceptable.

3) This question which mainly involved integral calculus was very well done by most candidates.

4) (i) The relevant value of χ^2 was usually found correctly. It was rare to find numerical errors.

(ii) The rule for calculating υ was often known but the answer was rarely clear. Many miscounted and thought there were 11 cells and 11 – 1=10. Since the answer was given the explanation needed to be clear.

(iii) Many scored high marks in the test. However, H_0 should have been *the data fits a Poisson distribution* rather than *the data fits a Po(3.87)*, as was often seen. Some candidates concluded the test with *there is evidence at the 10% significance level that a Poisson distribution is a good fit.* The significance level refers to rejection of H_0.

5) (i) Finding the median from the given CDF was mostly well done.

(ii) Many candidates have learned the relevant procedure and can distinguish between Y and y, Those that started with $G(y)=P(Y \le y)$ were often successful.

(iii) Many different methods were used to find $E(2-1/X^2)$ with many correct answers.

6) (i) Many managed the CI for the mean using a large sample.

(ii) The same variance as in Part (i) was often used which led to few marks.

(iii) This standard application of the difference between normal variables is very common but only the better candidates could manage it successfully.

(iv) Some candidates misread the question and gave more parts than (i).

7) (i) Most candidates realised that the two samples were not independent but were paired, hence a two-sample test was inappropriate.

(ii) Many candidates were aware that the difference in scores had to have a normal distribution but very few mentioned the randomness requirement.

Candidates were often successful with the test, but some used the two-sample test and scored very little.

(iii) It was hoped that candidates would use a significance level different to ½ %, but this was very rare.

4735 Statistics 4

General Comments

The entry was rather larger than last year's and the overall performance was similar, if slightly lower. The best-answered was Q4 on moment generating functions and the worst was Q7 on estimators, but even this scored well.

It was good to see that the two questions on hypothesis testing were well answered with conclusions not over-assertive. The procedures for the two Wilcoxon tests are fully described in the *List of Formulae*, so high marks can be obtained by careful candidates.

The presentation of candidates' work was usually good but some work on a question was often split over more that one page.

Comments on Individual Questions

1) (i) The p.g.f. of the B(n, p) was not often convincingly shown. The answer was given so care was needed in obtaining the result. Very often the last term of the relevant series was omitted.

(ii) The required algebra was mostly carried out successfully and this scored well.

2) (i) Some used the wrong test so scored very little. The hypotheses were usually given in terms of medians or median of differences, as is expected. We accept the use of m for population median but not μ without explanation.

(ii) Many candidates were aware that the rank sum test is for independent samples which was not the case in the question.

3) Many candidates scored full marks for this question, using several methods in part (i). Part (iii) required familiarity with conditional probability and also the expansion of $\mathrm{P}(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C})$; both were often correct.

4) (i) The procedure for finding E(X) from the m.g.f. was usually well known.

(ii) Most candidates knew that P(X=2) was the coefficient of e^{2t} in the expansion of the m.g.f. and this was usually correctly found. One candidate converted the m.g.f. to a p.g.f. and found $G''(0)/2$, which was easier than using the m.g.f.

(iii) Some candidates lost a mark by omitting the probability distribution of Y.

5) (i) The main advantage of a non-parametric test is that it does not require knowledge of the probability distribution of the breaking strengths and many knew this.

(ii) The test was usually accurately performed, but some used a normal approximation and were penalised if they did not use a continuity correction.

(iii) This proved to be less challenging than was expected and a majority scored well.

6) (i) Those familiar with S1 probability principles could obtain the necessary results.

(ii),(iii) These mostly scored full marks

(iv) Only a few candidates used an argument and even then they went on to perform a calculation. Most were aware that the dependence implied that $\text{Cov}(L,C) \neq 0$.

7) (i) Usually answered well with only a few errors seen in the variance.

(ii) Most candidates could obtain the result $a+b=1$, and some just stated it.
$\text{Var}(T)$ was minimised using a variety of methods but lost a mark if differentiation was used without justification of a minimum.

(iii) All knew that the estimator with smaller variance was better. This did not score if the relevant variances had not been obtained.

(iv) The variance of the sample mean was usually known and correctly quoted.

4736 Decision Mathematics 1

General Comments

Many candidates found this paper difficult, even in the early questions, although most were able to attempt at least some part of every question. Some candidates wasted time by copying out the Simplex tableau several times. However, good candidates performed well and answered the more challenging questions thoughtfully.

Candidates need to write their answers in the correct spaces in the answer book, and if additional sheets are used these should be labelled with the question part number. Candidates also need to be aware that writing over an answer can sometimes mean that it impossible to read the new attempt, it is preferable to strike out the original and write the new answer near it.

Comments on Individual Questions

1) (i) Most candidates could find at least one of the boundary equations, and several found all three inequalities that defined the feasible region. The most common errors were writing $x = 2y$ instead of $y = 2x$ and misreading the scale of the graph.

(ii) Most candidates found the coordinates by solving the equations, though even those who read them from the graph were usually able to achieve sufficiently accurate values to get the marks. Some candidates treated this as an integer programming question, which was not the case.

(iii) Many candidates realised that the value $m = 1$ was critical, although some dropped a minus sign and ended up with –1. Some candidates then just gave a specific instance of a value for which each point was optimal rather than giving a general (algebraic) range.

2) (i) Most candidates were able to obtain the iterates 8, 5, 4.1, 4.00 for R, a few stopped at 4.1 and some made arithmetic errors in applying the algorithm.

(ii) Most candidates got the values 1, 1.5, 1.42 (either using fractions or decimals), although some rounded 1.4167 to 1.41 rather than 1.42. Some candidates gave the final value as 1.42, but most carried on to achieve 1.41 (or a value that rounded to 1.41). A few candidates continued well past the point where the two values agreed to 2 decimal places, which was not penalised as they had already wasted time.

(iii) The algorithm is based on Heron's (or Hero's) method for finding square roots, and for positive inputs will eventually achieve convergence at the square root of N. Because of the rounding of R and S to 2 decimal places, this version of the algorithm achieves an approximation to the square root for positive inputs. We only required candidates to recognise that the two values they had found in parts (i) and (ii) were (close to) the square roots of the inputs.

(iv) Starting with $N = –4$ generates $R = –2$ and $S = 0$, the next value of R is then 0 and so S cannot be calculated because it would involve dividing by 0. The algorithm fails in this case because of a 'maths error' on the calculator. Of course, the input $N = 0$ (which does have a square root) would cause the same problem.

(v) Starting with $N = –2$ gives a chaotic sequence; we accepted any reasonable description of this idea or a list of at least the first four R values. Several candidates wrongly assumed that the algorithm would behave in the same way as in part (iv). The problem in this case can be avoided by building in a counter, where the algorithm could be terminated after, for example, ten executions of step 3.

3) (i) This should have been an easy mark; all that was required was to state that a graph cannot have an odd number of odd nodes, or that the sum of the orders must be even, which did not hold for this particular set of vertex orders.

Some candidates assumed that the graph needed to be simply connected and got involved in lengthy descriptions about why this could not be the case. Some candidates claimed that you cannot have a graph with an odd number of vertices, rather than with an odd total order.

(ii) The problem arises with the vertex of order five; in a simple graph with five vertices no vertex can be joined to more than four others. It is possible to draw a connected graph with these vertex orders.

There is an increasing trend for candidates to confuse the terms 'connected' and 'complete', resulting in some very peculiar explanations.

(iii) The sum of the vertex orders is twice the number of arcs, so this graph must have five arcs. A spanning tree with five vertices would only have four arcs.

(iv) Some candidates claimed that it was impossible to draw a simply connected semi-Eulerian graph that is also a tree; sometimes this was due to confusion over the term connected and sometimes because they assumed that trees had to branch. Any 'string' or non-branching tree will fit the required conditions. A variety of explanations were given for why it is impossible to draw a simply connected Eulerian graph that is also a tree, essentially the issue is that in an Eulerian graph every vertex has an even order whereas in a tree the 'ends' of the branches must have order 1. Some candidates gave good explanations of how the sum of the vertex orders for an Eulerian graph had to be at least ten, whereas a tree on five vertices has four arcs and hence total vertex order of only eight. The answer 'you cannot have a cycle in a tree' conjured up some interesting images but did get the mark.

(v) The graph needed to be either Eulerian or semi-Eulerian for such a route to exist, in this case it was neither since it had six odd vertices. To achieve a semi-Eulerian trail two arcs would need to be repeated (making four of the six vertices even and the other two still odd), so the minimum number of arcs that would have to be travelled twice was two.

4) (i) Most candidates were able to set up an appropriate initial Simplex tableau. Some, inevitably, had the wrong signs on the variables in the objective row (the objective row, in this case, corresponds to the equation $P + 3w - 5x + 7y - 2z = 0$). Some candidates omitted the slack variable columns, which was not penalised in this part but led to a loss of marks in parts (ii) and (iii). Those candidates who had a negative coefficient for x in the objective row were usually able to say why the pivot had to be chosen from this column, although some just claimed that it was because this column had the most negative values in the objective row without also showing why the z column could not be used.

(ii) Many candidates were able to carry out the first iteration correctly and read off the resulting values for the variables. When showing how each row was obtained it is acceptable to use notation such as '(1)+5pr', but not just '+5pr'. Some candidates did not know how to deal with the row where the entry in the pivot column was already 0.

Candidates should be aware that every tableau needs to have the appropriate number of basis columns, consisting of 0's and a single 1, that the values in the column representing the right-hand side of the equations should be non-negative and that the value of the objective should not decrease from one iteration to the next.

(iii) Several candidates achieved a successful second iteration and read off the resulting values for the variables. The problem represented had an unbounded feasible region and this showed up in the final tableau by there still being a negative entry in the objective row but there being no valid pivot choice. In the original LP formulation it can be seen that z can be increased without limit without violating the constraints, for example by setting w, x and y as 0, and hence P can be increased without limit. Some candidates claimed that x could also be increased without limit, but this is not true since $4w+5x+y$ is bounded above and z, y are both non-negative. Some candidates said that the problem was that there were four variables, apart from P, and only three constraints, however this argument on its own is not enough, for example the single constraint $-3w+5x-7y+2z \leq 10$ would have meant that P was bounded above.

5) (i) Dijkstra's algorithm was generally applied accurately, although some candidates made arithmetic errors. Some candidates wrote down extra temporary labels (such as 8.4 or 12.2 at F) or omitted temporary labels (such as 8.7 at F).

Most candidates wrote down the route of their shortest path although some omitted to give its length.

(ii) Many candidates either identified that the smallest value of x is 4.6 or gave the weight $4.4+x$, some candidates claimed that x had to be 4.7 or 4.5, presumably being confused over the inequality with continuous values.

(iii) Again, many candidates either identified the boundary values 3.5 and 1.8, respectively, or gave the weights $2.7+x$ and $2.3+x$, respectively. Some candidates rounded up or down in an inappropriate way.

(iv) The majority of the candidates calculated the time as being approximately 80 seconds. The most common error was to assume that the algorithm was linear, so doubling the size of the problem would only double the time needed.

6) (i) Many candidates achieved full marks in this part by stating that A and G needed to be odd so C and E needed to be paired to give a total weight of $29.9+x$ km.

(ii) There was an error in this part of the question; this was very unfortunate and we apologise for this. The marking scheme was adjusted to take account of this error and further steps were taken at the award to ensure that candidates were not penalised.

There were various ways to answer this question. Either the nodes A, C, E and G need to be paired or arcs AB and FG must be travelled twice so B, C, E and F need to be paired. The shortest routes between pairs of these needed to be found, including x in the case when $x<1.8$.

For $x \leq 1.8$, each way of pairing the four nodes gives the same shortest total leading to a shortest route that should have had length $34.3+2x$ km. For $x > 1.8$ the arc BC is used and the shortest pairing leads to a route that should have had length $36.1+x$ km.

Because of the omission of the repeated $AB+FG = 1.9$ km in the expressions on the question paper all reasonable attempts to apply the route inspection algorithm were marked, including crossed out work that had been replaced. However attempts that were just lists of specific cases were not usually given any credit as they did not demonstrate the use of the route inspection method. Answers in which x had been given a specific value also generally gained little credit.

(iii) Most candidates were able to draw the minimum spanning tree and find its total weight, apart from those who had given x a specific value. The required route must be longer because the minimum spanning tree is, by definition, shorter than the shortest cycle (and this is why the method for finding a lower bound for the travelling salesperson problem on a network on a complete graph works).

The application of the nearest neighbour strategy should have given the route *A-B-D-C-F-G* and then stalled because it has not yet visited *E*. Once every vertex has been visited we are allowed to have repeats on the way back to the start.

(iv) Most candidates were able to draw the minimum spanning tree and find it total weight.

The nearest neighbour strategy now gives the route *A-B-C-D-E-F-G* as a route from *A* to *G* with weight 14.3 km. Because the network is not built on a complete graph we may be able to use shortcuts to find a shorter route, and indeed replacing *E-F* with *E-D-F* does just this; of course any shortcut must still enable every node to be visited in travelling from *A* to *G*.

The upper bound for the original problem requires returning from *G* back to *A* as well. The shortest way to do this is to use the route *G-F-D-B-A* giving a total weight of 17+x km.

4737 Decision Mathematics 2

General Comments

The candidates for this paper were, in general, well prepared and were able to show what they knew. However, as in previous reports, candidates should be reminded to read the questions carefully as several dropped marks for not answering exactly what had been asked.

Centres should alert their candidates that work drawn on diagrams or tables in pencil, even if subsequently erased, shows up the same as work in ink. There was a problem with candidates who wrote over their original answers resulting in neither attempt being able to be read. It is preferable to strike out the original and write the new answer near it. Highlighter pen and gel pen rarely shows up on the scans and coloured pens all show up as being the same as black, with the exception of graphs which are colour scanned.

If additional sheets are used these should be labelled with the question part number.

Comments on Individual Questions

1) (i) Nearly all the candidates were able to draw the bipartite graph correctly.

(ii) Most candidates showed the incomplete matching correctly, although some also showed the arcs from the original bipartite graph or added the alternating path to their incomplete matching.

(iii) Some candidates had problems finding the shortest possible alternating path from 6 to *A*, often because they started at *A* instead of starting at 6.

Candidates were required to write down the matching and so diagrams were not accepted as being full solutions.

(iv) Most candidates were able to find this complete matching, although several did this without having written down the alternating path or by starting again from the original matching instead of using the incomplete matching from part (iii).

2) (i) Virtually all candidates were able to explain why the dummy columns were needed and most were able to explain how to modify the table to convert the problem to a minimisation. The dummies should not have positive scores because they should not be more suitable than any of the family members.

(ii) Most candidates followed the instructions to assign a cost of 0 to the dummy columns before modifying the matrix. The row and column reductions were usually carried out properly; only a few candidates used wrong methods. It is helpful if the different tables are briefly labelled to indicate what each represents.

Some candidates were not able to carry out the augmenting operations correctly; the most common error being to reduce uncrossed values by 4, for example, but only increase the values crossed through twice by 1. A few only augmented by 1 at a time.

Most candidates who successfully achieved a reduced cost matrix were able to give the complete matching. Some candidates read the rows and columns the wrong way round (essentially interchanging 'from' and 'to') and some had Amir giving a present to himself.

(iii) Most candidates were able to carry out the augmentation correctly and find the appropriate matching. Some candidates assumed that it did not matter which cards were assigned to the dummies, and so allocated *Q* to *B*, which was not optimal.

3) (i) Many candidates described why no magazine was dominant over all the others rather than why there was no dominance between any pair of magazines.

A few of the candidates claimed, incorrectly, that there was no dominance because all the entries are positive, or showed that neither type of weather was dominant.

(ii) Most candidates were able to find the play-safe strategies and to show that the game is unstable. Some candidates subtracted a constant from the table first, but this made no difference to the outcomes.

Several candidates found the row minima but did not state that the play-safe strategy was 'Activity Holidays' or '*A*'. Some candidates calculated a column maximin instead of column minimax (or the maximin of the negatives of the entries).

A few candidates lost the mark for showing that the game is unstable because they referred to the play-safe strategies being different rather than their values being different. The safest way to answer these questions is to write down the values of the row maximin and column minimax and show that they are different.

(iii) Most candidates were able to calculate the expected incomes correctly. A few made arithmetic errors and some interchanged the 0.4 and 0.6.

(iv) Most candidates were able to identify the appropriate pair of magazines, however a few only gave the choice for one or the other of these extreme cases.

(v) Most candidates gave a correct graph, although some of them chose peculiar scales. In particular, there is no reason to extend the p axis beyond the range 0 to 1, and it is helpful if this range covers more than half the width of the graph (instead of just one square as some candidates chose).

The use of the graph was non-standard but many candidates coped well with it. The earlier parts of the question were meant to help candidates to understand this last part.

4) (i) Most candidates gave diagrams where the precedences were correct but some did not use dummies to avoid labelling clashes. Dummies should be used to ensure that no two activities share a common start and also share a common end point.

Most candidates drew directed arcs, but without these it was very difficult to try to follow through the forward and backward passes.

There were fewer instances than in the past of large numbers of unnecessary extra dummies. Very few candidates tried to use activity on node, which has not been in the specification for some time now.

(ii) The passes were usually correct, apart from the odd numerical slip or candidates forgetting to deal with dummy activities correctly.

Most candidates listed the critical activities, although not always correctly, but some did not state the minimum project completion time.

(iii) Most of the resource histograms were correct, or very nearly correct. Some candidates did not label the vertical axis. There were a few candidates who drew graphs that were not histograms because they had 'holes' or activities hanging out over an empty space. Several candidates chose to label the activities, which was not required but could have been helpful to them in answering the remaining parts of the question.

(iv) Most candidates who attempted this part were able to identify that activity *D* needed to be delayed until after *B* had finished. Some candidates forgot that *B* precedes *D* and tried to delay *B*.

(v) Some candidates chose to show this on a diagram, which was not required but was quite an efficient way to show the order of the activities. Some amended their diagram from part (iii), which sometimes made it very difficult to work out which parts of the diagram were the answers to which parts of the question, and often resulted in candidates losing marks in part (iii). Some candidates achieved the correct time while having precedence violations and some put *I* and *J* alongside with *H* following them, resulting in an extra hour being taken.

5) (i) Apart from arithmetic errors, most candidates calculated the capacity of the cut correctly. A few tried to include the arcs where the potential flow was from *T* to *S*.

(ii) Nearly all candidates were able to explain why these arcs could not be full to capacity. Some misunderstood the idea of arcs being simultaneously full to capacity.

(iii) Most candidates showed a suitable flow, although some tried to use labelling procedure notation. The cut was rarely correct, most candidates see no difference between the maximum possible flow in an arc and the actual flow in that arc.

(iv) Some candidates were concerned about how to show their answers to parts (iv), (v) and (vi) all on one diagram. The majority of candidates were able to set up the labelling correctly, although a few had the labelling totally reversed.

The excess capacities should be in the original direction of flow and the potential backflows in the opposing direction, so that originally the arrows opposing the original direction should all show 0.

Some candidates did not reverse the direction of the flow in arc *BC*.

(v) Most candidates were able to show this augmentation correctly.

(vi) Candidates who had successfully answered parts (iv) and (v) were usually able to find a suitable flow here. Some did not write down the flow augmenting routes and several did not write down the maximum flow.

(vii) Candidates who understood that the flow in arc *BC* had been reversed from the original diagram were usually able to show a maximum flow of 3800 and some were able to find the appropriate cut and explain how this showed that the flow was maximal. Just claiming that such a cut exists, as some candidates did, was not enough.

6) Many excellent solutions, although some candidates worked forwards and some found a maximum weight route instead of a maximin.

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