RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4736:** Decision Mathematics 1

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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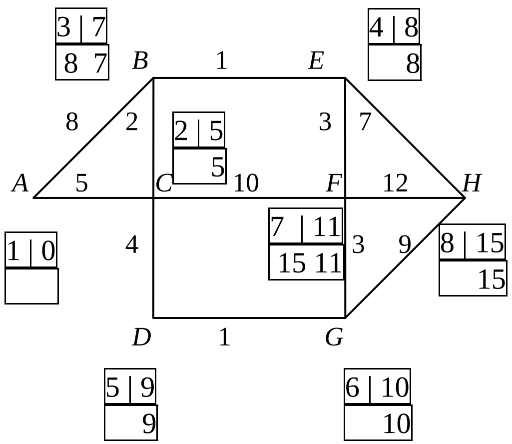
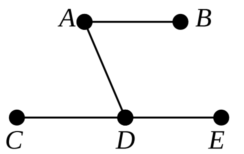
1	**(i)**		M1 A1		Any reasonable presentation of information Updating at *B* All temporary labels correct (and no extras)	Seeing 8 as a temporary label at *B* and 7 as a permanent label Not follow through
			M1		All permanent labels correct, cao (condone blank at *A*)	Not follow through
			A1		Order of labelling correct, cao	Not follow through
		Route: *A – C – B – E - H*	B1	**[5]**	cao – or in reverse	Not follow through
	(ii)	Odd nodes: *B*, *E*, *G*, *H*	B1		Odd nodes (may be implied from working)	Using *B*, *E*, *G*, *H* and no others
		BE + *GH* = 1 + 9 = 10 *BG* + *EH* = 7 + 7 = 14	M1		At least one correct total (10, 14, 14)	Correct method and value(s), not follow through
		BH + *EG* = 8 + 6 = 14	A1		All three pairings and correct totals seen	Both pairings (eg *BE*, *GH*) and totals, all correct
		Minimum is 10	B1	**[4]**	10 cao	Unsupported 10 gets B1
	(iii)	Need *D* and *H* odd, so need to consider pairings using *B*, *D*, *E*, *G*	B1		Seen or implied (without having to check route)	Do not use their route to deduce this, it could, however be seen from their pairings
		The minimum pairing is *BE* + *DG* = 1 + 1 = 2 (any other pairing must be longer)	B1		Repeat *BE* and *DG* <u>stated</u> (without having to check route)	Need to see *BE*, *DG* identified, not just 1+1=2
		A possible route is *DCABEHGDGFCBEFH*	B1	**[3]**	A possible route	15 letters, starting at *D* ending at *H* and repeating *BE* and *DG*

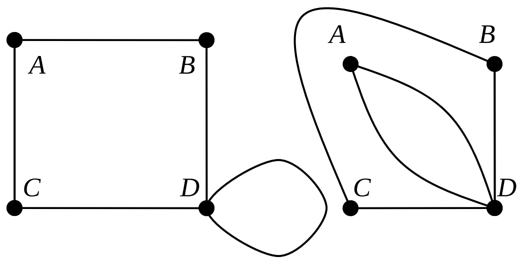
2 (i)

	A	*B*	*C*	*D*	*E*
A	-	~~12~~	~~30~~	~~15~~	~~22~~
B	**12**	-	~~24~~	~~16~~	~~30~~
C	30	24	-	**20**	25
D	**15**	~~16~~	~~20~~	-	~~10~~
E	~~22~~	~~30~~	~~25~~	**10**	-

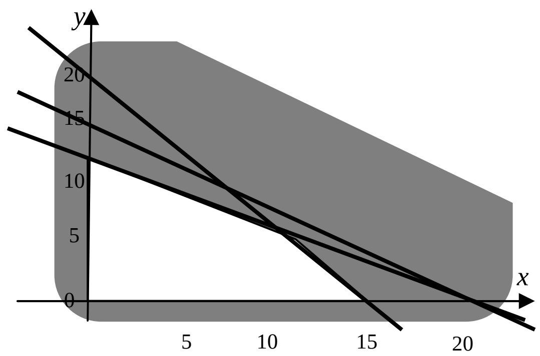
AB = 12
AD = 15
DE = 10
DC = 20



Question	Part	Answer	Marks	Total	Guidance	Notes
2	(i)	Matrix (above)	B1		Correct entries chosen on matrix (and no others)	Or with rows and columns interchanged throughout
		AB = 12, *AD* = 15, *DE* = 10, *DC* = 20	B1		Arcs chosen in correct order (written down)	Must be arcs not vertices (asked for in question)
		Tree (above)	B1		The correct tree, cao	Not follow through
		57 metres	B1	[4]	57 cao	Not follow through
	(ii)	Two shortest arcs from *F* = *BF* + *EF* = 29+30 = 59 57 + 59	M1		59 + their mst weight from (i)	Method mark may be implied from answer
		= 116	A1	[2]	116 cao	Not follow through
		A – *B* – *D* – *E* – *C* – *F* – *A*	M1		Applying nn to get *A* – *B* – *D* – *E* – *C*	Allow even if it stops at *C* or goes wrong after *C* Method mark may be implied from 126
		12 + 16 + 10 + 25 + 31 + 32 = 126	A1	[2]	126 cao	Not follow through

3	**(i)**	Cannot have an odd number of odd vertices (nodes) (Note: the question does not say that this graph has to be simply connected)	B1	**[1]**	Three odd nodes Must have an even number of odd nodes 1+2+3+3 =9 which would mean 4½ arcs	Not from a diagram of a specific case (and not from talking about what the vertices of order 3 connect to, for example) Not just ‘sum = 9’
	(ii)	Not simple Cannot have a vertex of order 4	B1	**[1]**	Identifying that the graph cannot be simple and an explanation that involves the vertex of order 4 Condone ‘not connected ... and not simple ...’ with a valid reason for the ‘not simple’ part	If the term ‘simple’ is not used the answer must talk about the vertex of order 4 forcing repeated arcs or loops (allow either) or equivalent
	(iii)	All nodes are even (and graph is connected)	B1		Vertex orders all even	2, 2, 2, 4 are all even
		eg eg 	M1		A labelled connected graph with four vertices *A*, *B*, *C*, *D* with orders 2, 2, 2, 4 respectively	Must be connected and labelled as well as having orders 2, 2, 2, 4
		eg *A*–*B*–*D*–*D*–*C*–*A* eg *A*–*D*–*C*–*B*–*D*-*A*	A1	**[3]**	A valid Eulerian trail for their graph, written down unambiguously (not just indicated on diagram)	May start at any vertex but must close the tour by finishing at the start vertex. May write as a list of arcs, directions not necessary
	(iv)(a)	a, b and c can only take the values 0, 1 or 2	B1		Condone ‘must be 1 or 2’, condone $0 \leq a, b, c \leq 2$ Must be less than 3	Do not accept < 2 or $1 \leq a, b, c \leq 2$
	(b)	None of a, b and c are zero	B1		‘Not 0’ or ‘all positive’ or equivalent Accept ‘one $\geq$ 2 and others $\geq$ 1’	Allow ‘must be 1 or 2’ (using (a) as well as (b)) Condone $1 \leq a, b, c \leq 2$
	(c)	Two must be odd and the other even	B1	**[3]**	Allow ‘two odd’	Not specific values ((using (a) and (b) as well as (c) gives 1, 2, 1. This does not get this mark)

4	(i)	In the first pass through bubble sort we compare the first value with the second and swap if the first is larger than the second. We then compare the value that is now second with the third value and swap if the second is larger than the third. We continue like this to the end of the list.	M1 A1		Must be describing what happens in the general case, not just using a specific numerical example Compare first value with second, swap if first is larger (allow 'compare first and second') Then compare second with third, and so on	Compare first pair and swap if needed If first is bigger than second swap them Describing moving along list (but not shuttling back), if any ambiguity do not give this mark
		At this point the largest value will be in the final position and we can ignore it in subsequent passes. In the second pass we start again by comparing the first and second values, but we now only need to sort the first *n*-1 values.	M1 A1		Final (largest) value is in correct position Start again but only using first *n*-1 values	Last value guaranteed Repeat but with final value already fixed
		We continue in this way until either we have a list of length 1 to sort or we have a pass in which no swaps were made.	B1	[5]	Accept 'until no more passes are possible' or 'stop when whole list has been considered' Allow 'until only one item left' or 'until no swaps' or 'until all have permanent labels' or equivalent	Not just 'stop when list is sorted' Not just 'all numbers are in correct places'
	(ii)	Start with: 3 10 8 2 6 11 After first pass: 3 8 2 6 10 11 After second pass: 3 2 6 8 10 11 After third pass: 2 3 6 8 10 11 After fourth pass: 2 3 6 8 10 11 May label before pass is made, which will look like five passes but is OK	M1 M1 A1	[3]	Result of each pass must be easily found, do not imply from muddled working 3 8 2 6 10 11 shown at end of 1st pass 2nd pass correct, follow through their list from 1st pass if possible Final list correct (cao) <u>and</u> exactly <u>four passes</u> used (depends on both method marks)	Misread rule (a single value miscopied or omitted from the list given in the question) will penalise the A mark only, but miscopying from one line of their working to the next could also lose one or both M marks
	(iii)	3 10 2 8 6 11	M1 A1	[2]	3 10 8 and 11 correct All correct, in correct order (cao)	In correct order of planks and cuts (could be vertical or with first at bottom line)
	(iv)	11 8 10 6 3 2	B1		All correct, in correct order (cao)	May also see 11 10 8 6 3 2
		Little waste from first two planks and a piece of length 18 feet from the third, which may be more useful than three medium length waste pieces	B1	[2]	Unused piece 18 feet, may be more useful than three shorter pieces (5ft, 6 ft and 9 ft) left over Little waste from first two planks	Referring to the lengths of the pieces left over Not 'it uses fewer cuts' (it doesn't, they both use six cuts), must have all six pieces
	(v)	11 6 3 10 8 2	B1		This cutting plan, planks in either order, pieces within planks in either order	Must have all six pieces
		Two planks and four cuts	B1	[2]	2 planks, 4 cuts or 2 planks each cut twice	Do not imply '2 planks', must be stated

5	(i)	x = number of parcels per hour from new customers y = number of parcels per hour from occasional customers z = number of parcels per hour from regular customers	B1	[1]	Accept identifying x with new, y with occasional and z with regular with reference to 'number of parcels per hour' and 'customers' missing or wrong Condone x = new, y = occasional, z = regular	Do not accept if x, y and z are not separately identified, unless order is unambiguous So, 'the number if parcels from the three types of customer' or 'number of new, occasional and regular parcels' are not enough, unless supported by words like 'in that order' or 'respectively'
	(ii)	Contents: $3x + 5y + 2z \leq 60$ Postage: $4x + 3y + 3z \leq 60$ Address: $3x + 4y + 3z \leq 60$ $x \geq 0, y \geq 0, z \geq 0$	B1 B1 B1 B1	[4]	cao need not have identified with contents, not < cao need not have identified with postage, not < cao need not have identified with address, not < cao	Allow use of slack variables (assume slack ≥ 0) and allow scaled versions, provided they are correct If slack variables have been used then these must also be identified as non-negative here
	(iii)	Can ignore the z term Objective function becomes $P = 8x + 7y$ Constraints become $3x + 5y \leq 60$ $4x + 3y \leq 60$ $3x + 4y \leq 60$ $x \geq 0, y \geq 0$	B1 B1	[2]	Saying that we can ignore z (or equivalent), or writing out the objective with z removed Writing out all their constraints with z removed (must have at least two linear constraints that involve both x and y)	Need not say 'Maximise' and may omit 'P =' Follow through their constraints Condone omission of non-negativity constraints
	(iv)	(20, 0) (0, 12) (15, 0) (0, 20) (20, 0) (0, 15) 	B1 M1 A1	[3]	Axes scaled and labelled appropriately Boundaries of all their constraints shown correctly, at least two linear constraints that involve both x and y, extending far enough for feasible region to plausibly be seen <u>Correct</u> graph with correct shading or feasible region correct and clearly identified (cao) Need not shade $x < 0$ and $y < 0$ May also show a profit line (eg joining (0,8) to (7,0) or (0, 16) to (14, 0))	x and y labels (and some scale markings on both) Lines joining (20, 0) to (0, 12); (15, 0) to (0, 20) and (20, 0) to (0, 15) or follow through theirs Tolerance $\pm$ 1 little square on axes Not follow through for A mark

		Checking P at (one or more of the) vertices of their feasible region (to nearest integer or better) <u>or</u> using a profit line (of negative gradient) (15, 0) gives $P = 120$ (10.9, 5.45) gives $P = 125.45$ (0, 12) gives $P = 84$ Check 10.9 parcels from new customers and 5.45 parcels from occasional customers on average each hour.	M1 A1 A1	 [3]	May be implied from <u>correct</u> answer (to nearest integer or better) Optimum point correct to nearest integer or better – accept (11, 5) or (11, 6), allow (10, 6) Giving ($\frac{120}{11}$, $\frac{60}{11}$) or (10 $\frac{10}{11}$, 5 $\frac{5}{11}$) or (10.9, 5.5) or (10.9, 5.4), or better, need not be in context	Correct vertex marked or answer 125 (or better) for optimum value or either of (11, 5) or (11, 6) (or better) given as optimum point implies M mark Following through their graph. Do not follow through to a different optimal vertex for the A marks Allow '10.9 new and 5.5 occasional' (or 5.4 or better) Allow '$x = 10.9$ and $y = 5.5$' (or 5.4, or better)
	(v)	x and y must now be integers (10, 6) gives $P = 122$ (11, 5) gives $P = 123$ (9, 6) gives $P = 114$ (12, 4) gives $P = 124$ (8, 7) gives $P = 113$ (13, 2) gives $P = 118$ (7, 7) gives $P = 105$ (14, 1) gives $P = 119$ (6, 8) gives $P = 104$ (15, 0) gives $P = 120$ and so on Check 12 parcels from new customers and 4 from occasional customers	B1 M1 A1	 [3]	Recognising that x and y must both be integers, or implied from answer – even if this is the same as the answer to part (iv) Testing feasible integer points or using a profit line on <u>integer</u> feasible points, may be implied from answer being given as one of (10, 6), (11, 5) or (12, 4) cao, need not be in context	Sufficient to give <u>any</u> integer point as final solution Sufficient to test one integer point in their feasible region Allow grid point dots on graph Accept '12 new and 4 occasional' or '$x = 12$, $y = 4$'
	(vi)	May not have enough parcels of each type Cannot do two checks at the same time on the same parcel	B1	[1]	Any valid reason	Not a criticism of the values for timings or points given in the question

<table>
<tr><td>6</td><td>(i)</td><td>$a = 6-x$, $b = 8-y$, $c = 10-z$

Minimise $2a - 4b + 5c - 30$
$\Rightarrow$ minimise $12 - 2x - 32 + 4y + 50 - 5z - 30$
$\Rightarrow$ minimise $-2x + 4y - 5z$
$\Rightarrow$ maximise $2x - 4y + 5z$ (given)

$3a + 2b - c \geq 10$
$\Rightarrow 3(6-x) + 2(8-y) - (10-z) \geq 10$
$\Rightarrow 3x + 2y - z \leq 14$ (given)

$-2a + 4c \leq 35$
$\Rightarrow -2(6-x) + 4(10-z) \leq 35 \Rightarrow 2x - 4z \leq 7$ (given)

$4a - b \leq 20$
$\Rightarrow 4(6-x) - (8-y) \leq 20 \Rightarrow -4x + y \leq 4$ (given)

$a \leq 6 \Rightarrow x \geq 0$, $b \leq 8 \Rightarrow y \geq 0$, $c \leq 10 \Rightarrow z \geq 0$</td><td>B1

M1

A1</td><td>[3]</td><td>Replacing a, b and c in objective to get $2x - 4y + 5z$

Replacing a, b and c in the first <u>three</u> constraints

to get the given expressions

Not necessary to show how $a \leq 6$, $b \leq 8$, $c \leq 10$ give $x \geq 0$, $y \geq 0$, $z \geq 0$</td><td>Evidence of $2(6-x) - 4(8-y) + 5(10-z)$, with or without -30 and with or without 'minimise'

Replacing a by $6-x$, b by $8-y$ and c by $10-z$ in <u>all three</u> constraints

Convincingly achieving the given expressions, including dealing with the inequality signs</td></tr>
<tr><td></td><td>(ii)</td><td>Tableau A (below)</td><td>M1

A1</td><td>[2]</td><td>Constraint rows correct, with three slack variable columns

Objective row correct</td><td>Condone P column missing
Rows and columns may appear in any order
Slack variable columns must consist of 0's and a 1

Not the negatives of these values (2 -4 5 0 0 0 0)</td></tr>
<tr><td></td><td></td><td>Tableau B (below)

New row 3 = (row 3) ÷ 2 (even if –ve pivot)
New row 1 = row 1 + 2(new row 3)
New row 2 = row 2 – 3(new row 3)
New row 4 = row 4 + 4(new row 3)

Pivot row method may be implied</td><td>M1

A1

B1 ft</td><td>[3]</td><td>An augmented tableau with four basis columns (or three with P column missing), non-negative values in final column and value of objective having not decreased

Correct tableau after one iteration (cao)

Method seen and correct, any reasonable form
Or: new row 1 = row 1 + original row 3
new row 2 = row 2 – 1.5 (original row 3)
new row 3 = row 3 ÷ 2
new row 4 = row 4 + 2(original row 3)</td><td>M mark is for any tableau that satisfies these conditions and is different from the original
Basis columns must consist of 0's and a 1

A mark is not follow through and requires a P col

May use 'row 3' to mean original or new row, provided consistent
eg for row 1 allow any of + 2r3, r1+2r3, +2pr, etc
or +r3, r1+r3, etc</td></tr>
</table>

Tableau A:

P	x	y	z	s	t	u	RHS
1	-2	4	-5	0	0	0	0
0	3	2	-1	1	0	0	14
0	2	0	-4	0	1	0	7
0	-4	1	0	0	0	1	4

Tableau B:

P	x	y	z	s	t	u	RHS
1	0	4	-9	0	1	0	7
0	0	2	5	1	-1.5	0	3.5
0	1	0	-2	0	0.5	0	3.5
0	0	1	-8	0	2	1	18

		Answer	Marks		Guidance	
		(tableau below)	M1		An augmented tableau with four basis columns (or three with P column missing), non-negative values in final column and value of objective having not decreased from 1st iteration	M mark is for any tableau that satisfies these conditions and is different from the original. Basis columns must consist of 0's and a 1
			A1		Correct tableau after two iterations (cao)	A mark is not follow through and requires a P col
		New row 2 = (row 2) ÷ 5 (even if –ve pivot) New row 1 = row 1 + 9(new row 2) New row 3 = row 3 + 2(new row 2) New row 4 = row 4 + 8(new row 2) Pivot row method may be implied	B1 ft	**[3]**	Method seen and correct, any reasonable forms Or: new row 1 = row 1 + 1.8(original row 2) new row 2 = row 2 ÷ 5 new row 3 = row 3 + 0.4(original row 2) new row 4 = row 4 + 1.6(original row 2)	May use 'row 3' to mean original or new row, provided consistent
		$x = 4.9, y = 0, z = 0.7 \Rightarrow a = 1.1, b = 8, c = 9.3$	B1		Correct values for a, b and c (cao)	Not follow through, not just the x, y and z values
		$2a - 4b + 5c - 30 = -13.3$	B1	**[2]**	-13.3 (cao)	Not +13.3

Tableau:

P	x	y	z	s	t	u	RHS
1	0	7.6	0	1.8	-1.7	0	13.3
0	0	0.4	1	0.2	-0.3	0	0.7
0	1	0.8	0	0.4	-0.1	0	4.9
0	0	4.2	0	1.6	-0.4	1	23.6

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